Aspects of Lifshitz Holography

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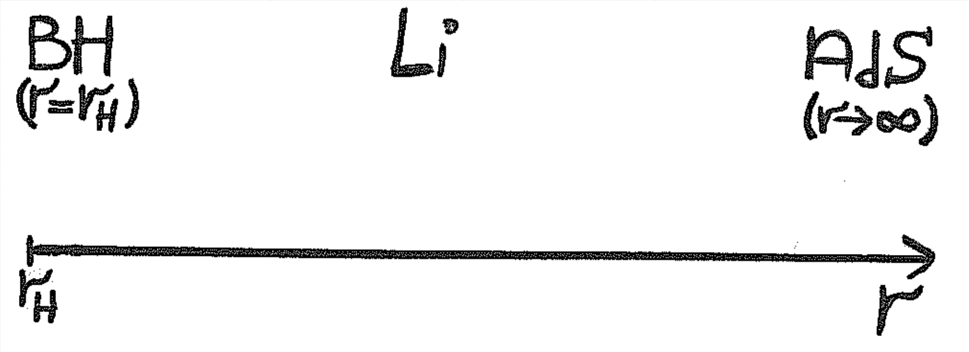
Overview

- AdS/CMT: anisotropic scaling symmetry in Li FT et al.
- Embedding Li in sugra/ST is hard, & fraught. We don't solve the deep problems, but take AdS asymptotics for UV-completion. Lifshitz-*like* gravity duals, not strict Li.
- 

- Dilaton gravity model & candidate gravity dual Ansatz
- Equations of motion & 3 crucial symmetries
- Limits: asympt. Li black branes & aAdS black branes
- Perturbative analytics about [flat] horizon & AdS
- Equation of state: $$\mathcal{E} = \frac{d}{d+1}(Ts + \mu n)$$
- Black Li/AdS numerics: no discont. phase transition
- How # FT degrees of freedom depends on z and d.

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Anisotropic field theory scaling

- 'AdS/CMT': seek candidate gravity duals describing desirable behaviour for quantum critical systems.
- Lifshitz field theory & others possesses anisotropic scaling symmetry. This affects time & space in (d+1) dim differently: $t \to \lambda^z t, \quad x_i \to \lambda x_i$
- Candidate gravity dual should possess this anisotropic scaling symmetry. By analogy with AdS (*z*=1), this suggests a metric of form (KLM: *z*=2)

$$ds^2 = L^2 \left(-r^{2z} dt^2 + r^2 dx^i dx^j \delta_{ij} + \frac{dr^2}{r^2} \right), \quad i \in \{1, 2...d\}$$

- where $\Lambda = -d(d+1)/(2L^2)$
- In gravity dual, radial coordinate must scale too:

$$r \to \lambda^{-1} r$$

- Candidate gravity duals to Lifshitz at finite *T* and μ will be "Li/AdS" black branes carrying charge.

The model and the gravity dual Ansatz

- Bulk action in (d+2) dimensions:

$$S = \frac{1}{16\pi G_{d+2}} \int d^{d+2}x\sqrt{-g}\left(R - 2\Lambda - 2(\nabla\phi)^2 - e^{2\alpha\phi}\mathcal{G}^2\right).$$

- Ansatz for candidate gravity dual:

$$ds^2 = -e^{2A(r)}dt^2 + e^{2B(r)}\left(dx^i dx^j \delta_{ij}\right) + e^{2C(r)}dr^2$$

$$\phi = \phi(r), \qquad \mathcal{A} = e^{G(r)}dt$$

- i.e., only radial coordinate dependence is permitted.
- Reduced 1-D action:

$$L_{1D} = \frac{d(d-1)}{2}e^{A+dB-C}(\partial B)^2 + e^{-A+dB-C+2G+2\alpha\phi}(\partial G)^2$$
$$+de^{A+dB-C}\partial A\partial B - e^{A+dB-C}(\partial\phi)^2 - e^{A+dB+C}\Lambda,$$

- Consistent truncation provided C EOM obeyed; C is Lagrange multiplier imposing Hamiltonian constraint.

EOM & 3 crucial symmetries

$$\frac{d(d-1)}{2}e^{A+dB-C}(\partial B)^2+e^{-A+dB-C+2G+2\alpha\phi}(\partial G)^2$$
$$+de^{A+dB-C}\partial A\partial B-e^{A+dB-C}(\partial\phi)^2+e^{A+dB+C}\Lambda=0,$$
$$d\left(2\partial\phi+\alpha\left(\partial A-\partial B\right)\right)e^{A+dB-C}=\boxed{D_0},$$
$$e^{A+dB-C}\partial\phi+\alpha e^{-A+dB-C+2G+2\alpha\phi}\partial G=\boxed{P_0},$$
$$e^{-A+dB-C+G+2\alpha\phi}\partial G=\boxed{Q}.$$

- 3 conserved quantities. 3 symmetries?
  - rescaling of time & space coordinates that leaves the FT (d+1)-volume invariant:
    $$(A,B,C,\phi,G)\to(A+d\delta_1,B-\delta_1,C,\phi,G+d\delta_1)$$
  - global part of U(1) gauge symmetry:
    $$e^G\to e^G+\delta_3$$
  - redefining gauge coupling by shifting the dilaton:
    $$(A,B,C,\phi,G)\to(A,B,C,\phi+\delta_2,G-\alpha\delta_2)$$

Limits: aAdS and aLi black branes

- aAdS black branes:

$$A(r) = \ln\left(Lr\sqrt{1-\left(\frac{r_h}{r}\right)^{d+1}}\right), \quad B(r) = \ln(Lr),$$

$\boxed{s \propto T^d}$ (scaling)

$$C(r) = \ln\left(\frac{L}{r\sqrt{1-\left(\frac{r_h}{r}\right)^{d+1}}}\right), \quad \phi(r) = \phi_b, \quad \mathcal{A} = g_b dt.$$

- a"Li-like" black branes (dilaton runs logarithmically):

$$A(r) = \ln\left(L_L a_L r^z\sqrt{1-\left(\frac{r_h}{r}\right)^{z+d}}\right), \quad B(r) = \ln(Lr),$$

$\boxed{s \propto T^{d/z}}$ (scaling)

$$C(r) = \ln\left(\frac{L_L}{r\sqrt{1-\left(\frac{r_h}{r}\right)^{z+d}}}\right), \quad 2\alpha\phi(r) = \ln\left(r^{-2d}\Phi\right),$$

$$\alpha = \sqrt{\frac{2d}{(z-1)}} \qquad G(r) = \ln\left(\frac{(z-1)L^d a_L r^{z+d}\left(1-\left(\frac{r_h}{r}\right)^{d+z}\right)}{2Q}\right).$$

Perturbing about horizon & AdS

- Perturb analytically out from the horizon, & in from AdS infinity. Then use numerics to connect them.
- Horizon:

$$e^{G_1} = a_0\left(g_0(r-r_h)+g_1(r-r_h)^2+\cdots\right)\,,$$
$$e^{A_1} = a_0\left((r-r_h)^{\frac{1}{2}}+a_1(r-r_h)^{\frac{3}{2}}+\cdots\right),$$
$$e^{C_1} = \frac{c_0}{(r-r_h)^{\frac{1}{2}}}+c_1(r-r_h)^{\frac{1}{2}}+\cdots$$

- (in $P_0=0$ gauge, B does not get perturbed.)
- AdS: metric, dilaton pert's contain $r^{-(d+1)}, r^{-2d}$ terms. Gauge perturbation contains $r^{-(d-1)}$.
- Constants in each pert depend on: $\alpha, d, r_h, D_0, Q, c_0$.
- The first two are theory parameters. Final four?
- Fix r_h. Use 2 remaining syms to set D_0, Q at will.
- So then c_0 parametrizes a family.

Thermodynamic equation of state

- From perturbation analytics about horizon read off

$$T = \frac{r_h^2 a_0}{4\pi L c_0} = \frac{D_0}{\alpha d r_h^d 2\pi L^{d+1}}, \qquad s = \frac{r_h^d}{4G_{d+2}}$$

- Number density & chemical potential are

$$n = \frac{Q}{4\pi G_{d+2} L^{d-1}} \qquad \mu = \frac{g_b}{L^2}$$

- Energy density $\mathcal{E}$? Use background subtraction with AdS (in Poincare patch) as reference spacetime. (Curvature of horizon is zero so no Casimir-y terms.)
- $$\mathcal{E} = \frac{1}{16\pi G_{d+2}} \frac{2}{(d+1)} \frac{(D_0 + 2d\alpha Q g_b)}{\alpha L^{d+1}}.$$
- The above matches with scaling arguments (!), giving

$$\mathcal{E} = \frac{d}{d+1}(Ts + \mu n)$$

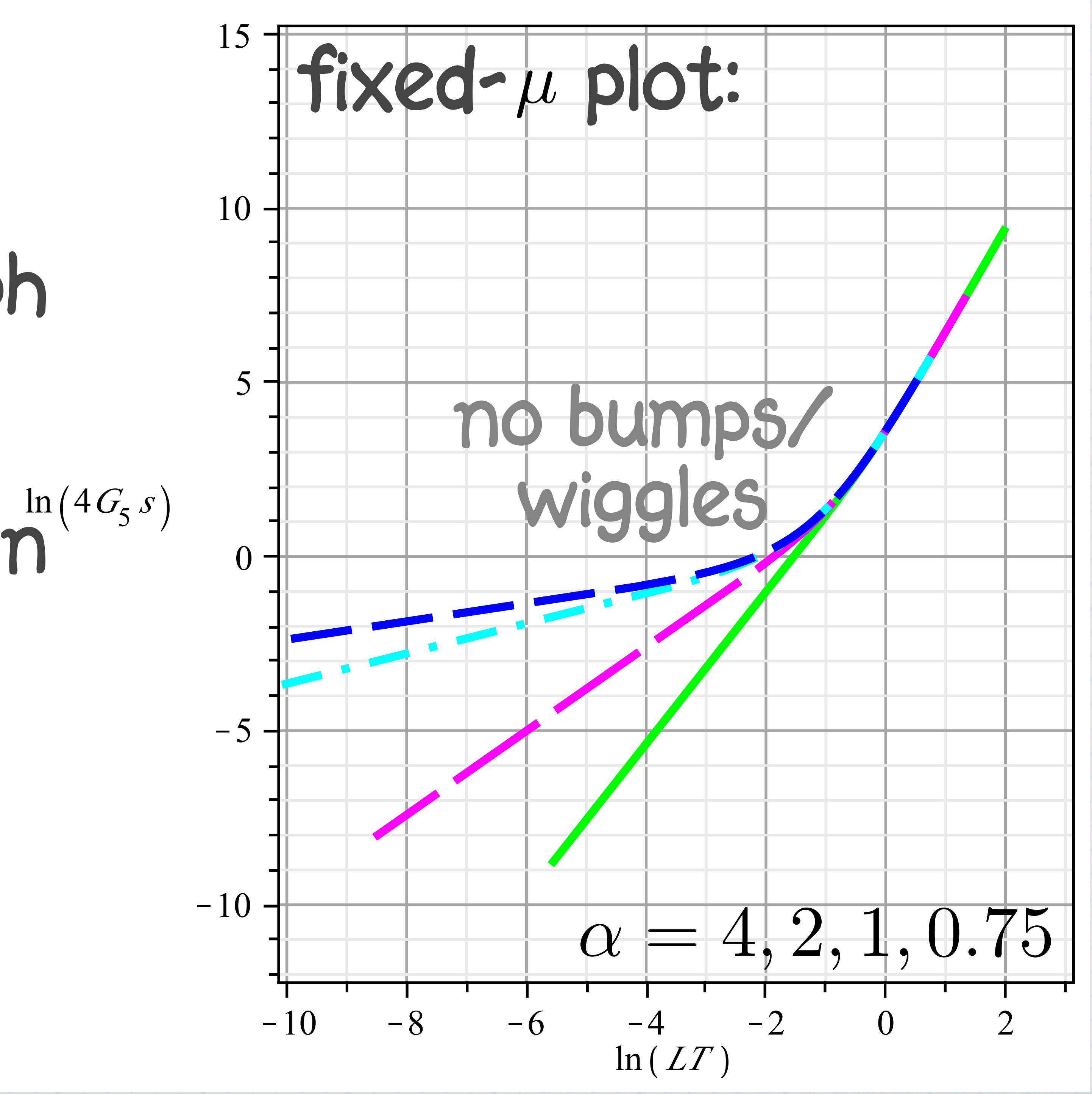
and for Li, $d/(d+z)$

Parametrizing Li/AdS black branes

- We can parametrize all our Li/AdS black branes by this one quantity c_0 that remained unfixed by symmetry.
- How does this work? Symmetries are key.
- The temperature is determined by D_0
- The chemical potential is controlled by g_b
- We work in $P_0 = 0$ gauge. Fix $r_h = 1$. Use 2 remaining symmetries to ensure that, when we numerically integrate out to infinity, we get *canonically* normalized metric functions for AdS.
- For any given horizon size, say $r_h = 1$, we can therefore map out solutions with various (T, μ).
- T/μ should occur only once for given horizon radius IF solutions with different param's are in fact unique.
- Limit $T \gg \mu$ is the close-to-AdS regime.
- Limit $T \ll \mu$ is the close-to-Li regime.

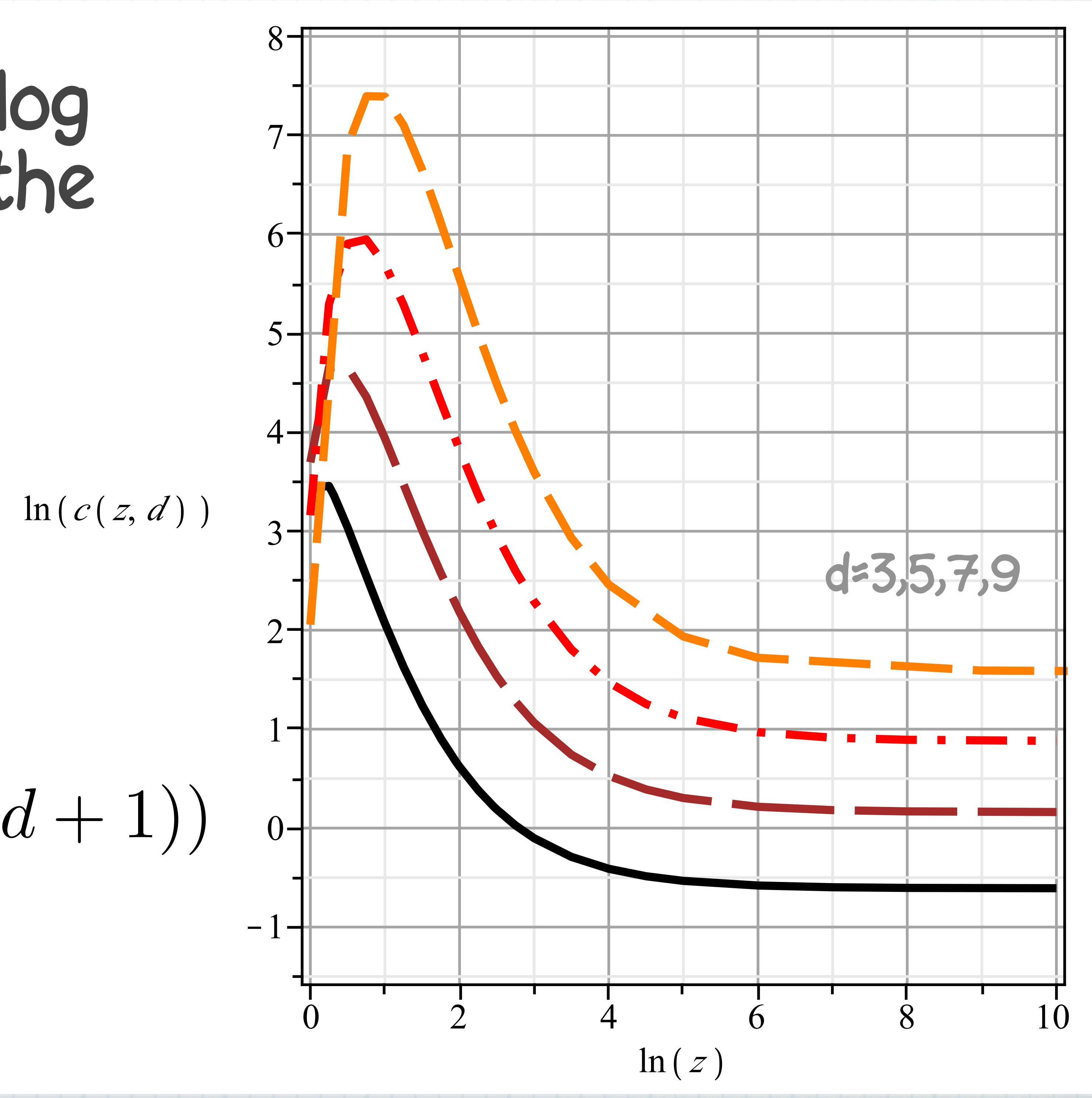
Black Li/AdS numerics

- Lifshitz black branes have $c_0 = \sqrt{\alpha^2 + 2}\sqrt{r_h/(d+1)}$
- AdS black branes have $c_0 = \sqrt{r_h/(d+1)}$
- Our Li/AdS solutions have a c_0 between these.
- So... is the graph of T/μ vs c_0 monotonic?
- Alternatively, is the graph entropy vs temperature monotonic?
- Yes. Smooth interpolation between Li-like scaling and AdS scaling.
- NO discontinuous phase transitions occur here for any d, z we studied.



Tracking Li degrees of freedom

- In Lifshitz-like regime, where $T \ll \mu$, we get $s = c(z,d)\,\frac{(L\mu)^d}{4G_{d+2}}\left(\frac{T}{\mu}\right)^{d/z}$
- Normalize dimensionful quantities using *L*, then plotting *s* vs *T* on a log-log plot gives *log(c(d,z))* as the intercept.
- *c* parametrizes # d.o.f.
- *c(d,z)* goes to constant at large *z* (!)
- In AdS (*z*=1) limit $T \gg \mu$ $s = [4\pi/(d+1)]^d (LT)^d$
- so $\ln(c(z,d)) = d\ln(4\pi/(d+1))$
- Explains small-*z* nonmonotonicity.



Scaling

- Tracking how degrees of freedom depend on d & z ?
- Doing dilute-gas calculation as rough estimate gives
$$s = \delta(z,d)^{-d/z}\kappa(z,d)\frac{\Omega_{d-1}}{(2\pi)^d 4G_{d+2}}\frac{(d+z)}{z^2}(L\mu)^d\Gamma\left(\frac{d}{z}\right)\left(\frac{T}{\mu}\right)^{d/z},$$
where $\delta\,,\kappa$ are *O(1)*. In the large-*z* limit,
$$\lim_{z\to\infty}\Gamma(d/z)(d+z)/z^2 = 1/d$$
- which is indep. of *z*, agreeing with numerics. So for
- $T \ll \mu$ get same scaling as gravity: $s = c(d)T^{d/z}\mu^{(z-1)d/z}$
- Li/AdS black brane numerics: no discontinuous phase transitions occur in going from $T \ll \mu$ to $T \gg \mu$.
- Thermodynamic equation of state:
$$\mathcal{E} = \frac{d}{d+1}(Ts + \mu n)$$
- Desired behaviour for AdS-embedded 'Li-like' dual.