Origin of AdS/CFT

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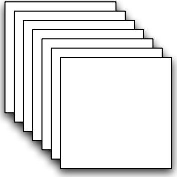
- AdS/CFT holography is an equivalence between gravitational (string) theories and *non*-gravitational field theories in one lower dimension. It gave the first concrete working models of the concept of holography for quantum gravity and grew out of studying nonperturbative D-branes in string theory.
- Consider a stack of N_p Dp-branes. Our key idea will be to have large N_p.



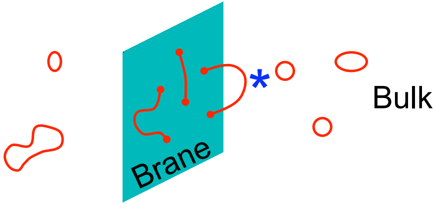
  To see why, consider how the gravitational warping of a single Dp-brane scales. Corrections to the flat Minkowski metric scale as

$$\delta g_{\mu\nu} \sim \frac{(G_N)(\tau_{Dp})}{r^{7-p}} \sim g_s^2 \ell_s^8 \frac{N_p}{g_s \ell_s^{p+1}} \frac{1}{r^{7-p}} \sim \frac{g_s N_p}{(r/\ell_s)^{7-p}}$$

  This shows that the variable controlling gravitational warping is $g_s N_p$.
- So we can obtain a seriously large warping of flat spacetime by having N_p become parametrically large while keeping g_s small.

Maldacena's decoupling limit

- Spacetime warping can be thought of as physics of closed strings. After all, the lowest state of excitation of a closed superstring includes the graviton mode. We can also look at the open string picture of Dp-brane physics.
- Lowest mode of open superstring has spin-one and mass zero. Theory living on worldvolume of N Dp-branes is $\text{SYM}_{p+1} + \alpha'$ corrections $+ g_s$ corrections. The variable controlling open string corrections when D-branes are present is actually $g_s N_p$, which is exactly what we found also controls spacetime warping.
- In general, dynamics of the full closed + open string sectors is unknown (c.f. string field theory). Maldacena's key insight was to take a very special 'decoupling limit' of this stack of N_p Dp-branes, which turns off interactions between closed strings in the bulk and open strings whose endpoints lie on the D-branes.



AdS=CFT

- In the decoupling limit, on the open string side you get just $d = p + 1$ SUSY Yang-Mills living on the Dp-brane worldvolume. On the closed string side, the decoupling limit zooms in on the *near-horizon* region of the geometry.
- Maldacena boldly realized that this decoupling limit meant that you could think about the stack of N_3 D3-branes *either* in terms of the near-horizon geometry *or* the worldvolume Yang-Mills theory. But because it is the *same* stack of D3-branes either way, those representations must be equivalent!
- The near-core geometries of D3, M2, M5, and D1-D5 systems produce $AdS_{p+2} \times S^{D-p-2}$ spacetimes, where D is the spacetime dimension, 10 in superstring theory and 11 in M theory. You will see how this works in HW4. In brief: the S^5 factor arises for D3 because the r^2 in the transverse piece of the metric proportional to $d\Omega_5^2$ is cancelled off by the $H_3^{+1/2}$ factor, because for D3-branes the harmonic function scales as $1/r^4$ in the near-core region.
- Yang-Mills theory on flat D3-branes has maximal SUSY, i.e. 16 supercharges. In $d = 4$ language this translates into $\mathcal{N} = 4$ SUSY. The supermultiplet has adjoint fermions and scalars partnering up with the adjoint vector field. The field content of this theory is so special that it ends up having zero β-function. It is actually a *conformal* field theory. Hence, we say AdS=CFT.

Probes and IR/UV relations

- AdS/CFT gives a definition of quantum gravity in asymptotically AdS space in *Lorentzian* signature, via the duality and Wick rotation in the CFT. AdS/CFT sheds light on the question of background independence in quantum gravity. It is independent of bulk background except for its asymptotics which are locked down by boundary physics. Not yet known how to do a worldsheet analysis for string theory in AdS, because a Ramond-Ramond flux is turned on. (c.f. v.technical Pure Spinor formalism of N.Berkovits, which is v.different to the NSR and GS formalisms.)
- One of the key ideas string theorists use every day is the concept of *probes*. Probes are objects that exist in the theory and are used to interrogate the physics. For fundamental string probes stretched between Dp-branes, field theory energy E is proportional to distance r between D-branes in bulk,

$$E = \frac{r}{2\pi\alpha'} \, .$$

  This is known as an *IR/UV relation* and is central to AdS/CFT. It says that the UV in the field theory is welded to the IR in the bulk and vice versa.
- Polchinski and I showed that gravitons have different IR/UV relations. Still, how deep it penetrates into the bulk scales inversely with energy on the brane.

IMSY and applications of AdS/CFT

- For more general Dp-branes, you find that the harmonic functions conspire to *not* give you $AdS_{p+2} \times S^{D-p-2}$ spacetimes. Instead [for $p < 5$], you get a warped version where there is an r- (and p-) dependent conformal factor. What does this mean for the possibility of a correspondence like AdS/CFT? The 10D bulk spacetime background is only reliable for certain ranges of r, because either $\alpha'\mathcal{R}$ or e^{Φ} is monotonically growing as $r \to 0$ or $r \to \infty$. By dimensional analysis, the field theory coupling also runs with energy. 10D SUGRA geometry becomes unreliable when $g^2_{\rm eff}(E) := g_s N(\ell_s E)^{p-3}$ becomes of order unity, which is where the SYM field theory description takes over.
- This gives rise to the idea of a *phase diagram* for a D-brane/string system, which turns out to be a very generic and physically rich feature in holographic setups. There can be more than one description of the physics, but only one of them at a time can be weakly coupled.
- AdS/CFT has been applied to modelling quark-gluon plasma and cond-mat, with limited successes. Can calculate transport properties; for cond-mat the physics is quite different in spirit to the story of quasiparticles. Also inspired the study of dS/CFT correspondence where the CFT is non-unitary.
- H.Nastase review of AdS/CFT basics for beginners: 0712.0689.

Basic holographic dictionary

Holographic dictionary

- AdS/CFT is a strong/weak duality,

$$\frac{L_{AdS}^4}{\ell_s^4} \leftrightarrow g_{\mathrm{YM}}^2 N \,, \qquad g_s \leftrightarrow \frac{1}{N} \,.$$

  This is why it is so powerful: to learn about strongly coupled CFT we use weakly coupled string theory in AdS and vice versa.
- Isometries of the bulk match up with symmetries of the CFT.
  1. The S^5 factor has a natural $SO(6)$ isometry. This is is also the R-symmetry of the $\mathcal{N}=4$ SUSY SYM theory.
  2. The AdS_5 factor has isometry group $SO(4,2)$. This is the conformal symmetry group of $\mathcal{N}=4$ $d=3+1$ SYM. $SO(3,1)$, the Lorentz group, is a subgroup.
- One of the reasons why AdS/CFT works is that near infinity, the area element grows like the volume element. The dimensionful constant of proportionality is furnished by the radius of curvature of AdS.
- Another very useful physics fact to know about AdS is that higher-ℓ partial waves do *not* fall off with higher powers of $1/r$ like they do in asymptotically Minkowski space. Influences deep in the AdS bulk can be easily seen from infinity. This is related to the fact that AdS has a timelike boundary.

Fefferman-Graham

- Maldacena derived AdS/CFT for the gauge theory on the worldvolume living on $\mathbb{R}^p$. Technically, the spacetime obtained was the *Poincaré patch* of AdS. Witten taught us that the correspondence actually extends to *global* AdS, which is dual to field theory living on S^p. Much beautiful physics derived.
- In asymptotically AdS, Fefferman-Graham coordinates are very handy,

$$ds^2 = \frac{L^2}{z^2}\left(dz^2 + g_{ab}(z,\vec{x})dx^a dx^b\right) ,$$

where L is the AdS scale, and $z = 1/r \in [0,\infty)$. $g_{ab}(z,\vec{x})$ is the boundary metric which obeys an expansion in powers of z/L. It is only fixed up to conformal transformation. Other bulk fields obey a similar near-boundary expansion. In general, get an overall power of z/L times a power law expansion with some logarithmic terms.
- Bulk EOMs are 2nd order PDEs, which yield two independent solutions. Working through the Fefferman-Graham details shows that coefficients of power law and log terms are all determined in terms of the first nontrivial coefficient function of $\vec{x}$. The key fact is that this boundary value of the bulk field, up to an overall power of z/L, is interpreted as a source term for a dual operator in the boundary field theory.

BF bound and GKP/W

- Example: scalar of mass m. Get

$$\phi(r,\vec{x}) = (z/L)^{d-\Delta}\left[\{\phi_{(0)}(\vec{x}) + \left(\tfrac{z}{L}\right)\phi_{(1)}(\vec{x}) + \cdots\} + (z/L)^{2\Delta-d}\{\phi_{(2\Delta-d)}(\vec{x}) + \cdots\}\right] ,$$

  where Δ solves $m^2L^2 - \Delta(\Delta-d) = 0$, giving

$$\Delta = d/2 \pm \sqrt{(d/2)^2 + m^2L^2} .$$

  Δ will turn out to be the scaling dimension of the dual operator in the CFT. Requiring $\Delta \in \mathbb{R}$ gives

$$m^2 \geq -(d/2L)^2 .$$

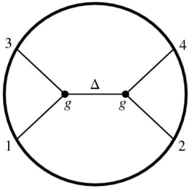
  This is the Breitenlohner-Freedman bound famous from SUGRA. Scalar fields in AdS can be a little bit tachyonic, but not too much, or AdS goes unstable.
- Gubser-Klebanov-Polyakov + Witten took this further, conjecturing

$$Z_{\rm string}\left[z^{\Delta-d}\phi(x,z)|_{z=0} = \phi_{(0)}(x)\right] = \langle e^{-S+\int d^dx\phi_{(0)}\mathcal{O}(x)}\rangle_{\rm CFT}$$

  Prime example of operator dictionary: $g_{\mu\nu}$(bulk) $\leftrightarrow$ $T_{\mu\nu}$(boundary). Non-normalizable modes: $\leftrightarrow$ turn on $\Delta\mathcal{L}_{\rm CFT} = \alpha_\phi\mathcal{O}_\phi$ in boundary (irrelevant ops). Normalizable modes: $\leftrightarrow$ turn on VEV $\langle\mathcal{O}_\phi\rangle = \beta_\phi$ (relevant ops).

Holography and black holes

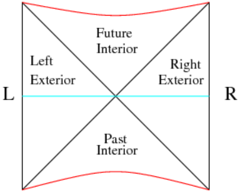
- Nonlocal probes: correlation functions, Wilson loops, entanglement entropy. Witten diagrams: bulk-boundary propagators and bulk vertices.



- UV/IR relations taught us that high-energy in CFT corresponds to near-boundary in bulk. Holographic RG: running understood via bulk Hamilton-Jacobi, including counterterms. *Flows* change interior of AdS.
- A very deep fact about AdS/CFT is that it is not just a zero-temperature equivalence. Asymptotically AdS black holes correspond to turning on finite temperature in the CFT. The Hawking-Page transition from BH to hot AdS is dual to the deconfinement transition in the boundary theory.
- AdS/CFT shows us how one extra bulk spacelike dimension emerges, but generically without a path integral proof as yet. It also does not show us how time emerges. That may require going beyond QM as we currently know it.

Maldacena's eternal AdS BH

- Maldacena also proposed in hep-th/0106112 that the *eternal* AdS black hole is dual to a direct product of two uncoupled conformal field theories, $CFT_L \times CFT_R$, with thermal entanglement between the L and R CFTs. Key feature: if you trace over either the left or right set of degrees of freedom, then you obtain a thermal density matrix at the Hawking temperature.
- In the holography literature this is known as the 'two-sided' black hole. Black holes formed in gravitational collapse are fundamentally different: they do not have a white hole singularity or a past event horizon and are called 'one-sided'.



- Maldacena's idea rests on an older construction by Werner Israel, a very famous Canadian gravity theorist, called the *thermofield double* or TFD state,

$$|\mathrm{TFD}\rangle = (1/\sqrt{Z})\sum_i e^{-\beta E/2}|\psi\rangle_L \times |\psi\rangle_R \,.$$

Advanced holography

Less symmetric holography

- To use holography to model real-world systems, need to break increasing degrees of SUSY and other symmetries. Big literature on this, divided into quark-gluon plasma modelling, AdS/condensed matter; also, dS/CFT.
- Fixing the asymptotics does not prevent you from having interesting phase transitions originating in interesting hair on the bulk solutions. There are much bigger, wilder classes of geometries available than previously imagined.
- Holography is applicable to systems other than $\mathcal{N} = 4$ SYM. Sometimes in a bottom-up setup we do not know what the dual QFT is, but we can still use holography to discern universal aspects of strongly coupled systems.
- Breaking boost: can get residual Schrödinger or Lifshitz symmetry. Breaking anisotropy and homogeneity. Modelling superconductors, glasses, strange metals, Fermi surfaces, hyperscaling violation, disorder. Holographic lattices.
- My interest in holography is more bottom-up style than top-down. Recent(ish) papers were on Lifshitz black holes [Bertoldi-Burrington-Peet] and on hyperscaling violating crossovers [O'Keeffe-Peet]. Current preprint on holographic modelling of disorder almost ready for posting [O'Keeffe-Peet].
- Neat part about holography from the point of view of someone interested in gravity: *geometrizing* phases of the dual QFT.

Higher-spin/vector holography

- Higher-spin theory is a lab that may provide a bridge between classical gravity and full quantum ST. Tower of modes. (c.f. ABJM, multi-M2, localization.)
- Old theorems had put stringent constraints on low-E scattering in flat spacetime that forbid $m = 0$ particles with spins $s > 2$ from participating in any interacting QFT. But in AdS, $\Lambda < 0$ provides dimensional coupling and IR cutoff, which can reconcile HS gauge symmetry w equivalence principle, giving M.Vasiliev's nonlinear unfolded equations of motion. Review: 1404.1948.
- J.Maldacena-A.Zhibodaev 1112.1016 showed why for 3D CFT with higher-spin symmetry you get a free theory, found for the dual of Vasiliev higher-spin theory in AdS_4. S.Giombi-X.Yin AdS_4 VH review: 1208.4036.
- R.Gokapumar-M.Gaberdiel 1207.6697 found the dual for AdS_3: a minimal model coset CFT_2 with $\mathcal{W}_N$ symmetry at large-N. (Scalar accompanying graviton and HS fields in 2D GG duality is massive; for AdS_4 it is massless.)
- GG actually got even further: they found that higher-spin in AdS_3 is a subsector of string theory, in the tensionless limit. Technically important.
- Higher-spin theory in 3D has BHs 1208.5182. Existence of horizons and singularities is not invariant under HS gauge transformations, but can define via *holonomy* if use Chern-Simons formulation of 3D gravity, e.g. 1302.0816.

Bulk locality

- AdS/CFT is a fascinating laboratory for studying process of thermalization. Study quenches, try to extract universalities. Review: 1103.2683.
  (Strong time dependence is harder than weak or none.)
  See also fluid/gravity correspondence; review by Hubeny: 1501.00007.
- (A.Hamilton-)D.Kabat-G.Lifschytz-D.Lowe showed in hep-th/0506118, hep-th/0606141, 1102.2910 how local operators in the AdS bulk can be represented via *smeared* operators in the CFT. Only regions in the causally relevant zone contribute. Their construction can be obstructed if there are bulk normal modes with exponentially small near-boundary imprint, such as for the AdS black hole 1304.6821. Is bulk locality emergent?
- M.vanRaamsdonk conjectured 0907.2939, 1005.3035 that smooth connected patches of geometry emerge from entanglement of regions on the boundary.
- Entanglement may not be enough to fully probe bulk geometry, esp. if BH. 1406.5859 by BCCdB discussed entanglement shadows and entwinement. Key Q: how much information can you ever reconstruct from the boundary?
- A.Almheiri-X.Dong-D.Harlow 1411.7041 argued that localization of bulk information should be understood in terms of quantum error correction. E.Mintun-J.Polchinski-V.Rosenhaus 1501.06577 connected this to boundary gauge invariance, suggesting it is closely connected to spacetime emergence.

Geometrization of entanglement

- S.Ryu-T.Takayanagi conjecture hep-th/0603001 relates the entanglement entropy $S_{\rm Ent}$ associated to a region R in the field theory to the area of the minimal surface in the bulk whose boundary is R. The holographic RT formula is important because it connects a geometrical bulk computation with an information theoretic field theory computation. Reviews: T.Nishioka-S.Ryu-T.Takayanagi 0905.0932, Headrick 1312.6717.
- An explanation of the RT formula was provided by A.Lewkowycz-J.Maldacena in 1304.4926, using a bulk version of the replica trick.
- 'Hole-ography' method computes entanglement for a hole in AdS spacetime 1310.4204 V.Balasubramanian-B.Chowdhury-B.Czech-J.deBoer-M.Heller. Uses differencing of RT formula, residual entropy.
- N.Lashkari-J.Simon in 1402.4829 argued that emergence of an effective notion of spacetime locality originates in restricting to a subset of observables unable to resolve black hole microstates from the maximally entangled state.
- People e.g. 1312.7856 also found that the first law for $S_{\rm Ent}$ – for small perturbations about CFT vacuum states, for ball-shaped regions – translates in the bulk to satisfaction of equations of motion linearized about AdS! Constraining the nonlinear story: 1405.3743. Also, entanglement inequalities can be used to derive conditions on bulk $T_{\mu\nu}$: 1412.3514.

Strings 2014 conference

Strings 2014

http://physics.princeton.edu/strings2014/

Broad list of topics featured:-

- **Mon** renormalization in ST, CFT bootstrap, instability of AdS, astroparticle experiments, string inflation
- **Tue** firewalls, holographic entanglement and geometry, entropy bounds, entropy inequalities and QFT, fuzzballs, higher-spin
- **Wed** holographic modelling of condensed matter, nonperturbative aspects of ABJM, topological aspects of SUSY gauge theories
- **Thu** aspects of SUSY gauge theories, gravity duals of higher-D SCFTs, mathematical ST, scattering amplitude technology in QFT and ST
- **Fri** mathematical aspects of QFTs and STs; five vision talks
- ***** poster session Mon, parallel sessions Wed, gong show Thu

Strominger poll: spacetime emergence

- Strominger polled dozens of us beforehand asking what we think are the key questions about string theory as a theory of quantum gravity. A selection:-

Schwarz Is there a formulation of M theory without reference to space or time? If so, what is the role of quantum entanglement?

Seiberg What is time and how does QM emerge? Given that space is emergent so should time be. Since it is hard to formulate QM without time, QM itself should be an emergent theory.

Sen What is the precise relation between quantum entanglement and classical geometry?

Freedman How much further can the present hints of a relation between entanglement and dynamical gravity be developed?

Ooguri How can one tell if a low energy effective theory cannot be completed as a consistent quantum theory of gravity? The gravity as the weakest force conjecture is a good example, but could one derive or find counter-examples to other folklores motivated by string theory?

Susskind What are the big principles we were missing whose absence prevents us from giving a definitive resolution? Answers will come from the connections between gravity and quantum information theory, in particular entanglement theory and computational complexity.

Strominger poll: holography and higher-spin

Kiritsis Is there a way to prove the QFT/string theory correspondence?

Takayanagi Is there any entropic meaning of internal spaces in AdS/CFT, e.g. S^5 in $AdS_5 \times S^5$, and how do they emerge holographically?

Erdmenger Using gauge/gravity duality, can we make predictions for universal properties of further observables, e.g. those currently measured in heavy-ion collisions at LHC, or in graphene and Dirac/Weyl semimetals?

Gaberdiel What is quantum gravity in 3D?

Vasiliev What is the exact relation between higher-spin gauge theory and string theory?

Giombi Do higher spin gauge theories define consistent theories of quantum gravity in general spacetime dimensions? If so, are they always related to some string theory?

Maldacena What is general theory of weakly coupled, interacting, higher spin particles? Is string theory the only solution, like GR is only solution of a similar Q involving massless spin-2 particles and leading order in ∂s?

Rastelli What are the universal constraints from symmetries, analyticity, and unitarity on the S matrix for quantum gravity?

Berkovits Can twistors be used to simplify superstring theory in a manner similar to its simplication of $\mathcal{N} = 4, d = 4$ SYM?

Strominger poll: formal aspects

Gaiotto How should we define string perturbation theory in Ramond backgrounds with string-scale curvature? The pure spinor formalism works in any consistent SUGRA background.

Staudacher Will we be able to use integrability in AdS/CFT to understand the precise mechanism how strings make gauge fields and vice versa? Will this mechanism contain generic elements, which apply to the host of non-integrable instances of gauge string dualities?

Dabholkar How to compute the quantum effects in bulk gravity in AdS/CFT holography and what can we learn from them about quantum gravity?

Gubser What is the quantum theory of N M2-branes? BFSS matrix theory and ABJM theory give good handles in special limits, but the full story is unknown.

Vafa Can we develop a perturbative computational scheme for 6D CFTs?

Rastelli What is the complete list of (super)conformal field theories in various dimensions, with different amounts of supersymmetry?

Kachru Can we prove new theorems in geometry and new constraints on consistent theories of gravity using techniques of 2D CFT? Constraints of modularity, the bootstrap, and AdS_3/CFT_2 combined give one a lot of possible levers which are far from fully exploited.

Strominger poll: black holes and cosmology

Polchinski	Do old black holes have smooth interiors?
Silverstein	At what level does perturbative string theory lead to a breakdown of effective field theory in time dependent systems such as naturally formed, evaporating BH? Does it generate sufficient non-adiabatic effects in BH physics to address the firewall problem?
anon.	Why is the entropy of most BHs captured by a Cardy-like formula?
Warner	Are there large deviations from the predictions of GR at the horizon scale of the kind of black hole that lies in the cores of some galaxies: near-extreme Kerr black holes with millions of solar masses?
Bousso	What is quantum gravity in a spacetime without boundary, such as a closed universe?
Raamsdonk	What is the Hilbert space/entanglement structure of a quantum gravity state representing a cosmological spacetime, e.g. de Sitter or an eternally inflating multiverse?
Larsen	What is the microscopic origin of de Sitter entropy?
deBoer	Does the nonlocality of quantum gravity have any implications for cosmology?
Kutasov	Is there a relation in string theory between SUSY breaking and cosmic acceleration?