Hiding extra dimensions

Essential problem we face in building real-world models: taking our string theory action principle defining the UV physics and *flowing down* to the phenomenologically relevant IR. This is generically messy and hard.

$\beta$-functions tell us that our 10D spacetime $M_{10}$ must obey the string theory equations of motion. String theorists commonly take a direct product (KK) ansatz $M_{10} = M_4 \times K_6$, where $K_6$ is some compact six-manifold. The details of what theory emerges on $M_4$ depends on the details of the manifold $K_6$ – not every 6-manifold is a solution. The low-energy theory in 4D that you get also depends on which superstring theory you chose: the precise structure of the 10D SUGRA field equations (the antisymmetric tensor fields especially) depends delicately on whether you have IIA, IIB, I, HE, or HO theory.

One challenge in model-building is getting the right spectrum of gauge forces and quark/lepton matter. Before the advent of D-branes, a no-go theorem prevented compactifications of IIA or IIB producing chiral fermions in 4D – this is why heterotic string theory was so popular in the first superstring revolution.

One of the most difficult aspects of building a credible compactification scenario is stabilizing all of the *moduli*, scalar fields which do not develop a potential to any order in perturbation theory but which we know must be absent from the low-energy massless spectrum. Recruit nonperturbative physics to fix.
Both the geometry and topology of $K_6$ play an important role in what low-energy physics we end up getting on $\mathcal{M}_4$. This originates in the fact that the 10D string theory worldsheet $\beta$-function equations are very picky about the spacetimes on which strings can propagate – if we make a product space ansatz it must be compatible with the field equations.

In particular, the number of light generations of fermion fields depends sensitively on the holonomy of the Calabi-Yau in KK compactifications. In the first superstring revolution, we discovered how to explain 3 generations in terms of the mathematics of one special type of Calabi-Yau.

Why Calabi-Yaus? These are manifolds with special holonomy which support the existence of Killing spinors, allowing SUSY to be present in the 4D theory. SUSY is not an observed low-energy symmetry in Nature (so far), but it is technically important in controlling UV physics. SUSY should be at most $\mathcal{N} = 1$; models with $\mathcal{N} \geq 2$ have unrealistic spectra.

SUSY introduces at least one extra scale in the problem (masses of superpartners), and this can actually permit Grand Unification. Just extrapolating Standard Model gauge couplings up to higher energy scales does not yield a GUT; this was proved experimentally via precision electroweak measurements at LEP in the tunnels now occupied by LHC.
Brane world models

Alternatives to Calabi-Yau compactifications of heterotic string theories?

Use Type IIA/IIB and add D-branes, fluxes, and orientifolds, which are objects which possess both a negative charge and a negative tension. N.B.: orientifolds do not destabilize the vacuum because they are fixed planes of a symmetry: they cannot fluctuate physically. So their negative tension is harmless. The physically crucial thing is that these negative tension objects which are fundamentally string theoretic let you evade previous no-go theorems which prevented building de Sitter compactifications in string theory.

Brane world idea # 1: we ‘effectively’ compactify the physics using a brane world (e.g. Randall-Sundrum) type model. These have a warped product space structure, in which the overall scale of the 4D geometry depends on the coordinate in the compact dimension. For example, in the RS models the bulk has $\Lambda < 0$ while the brane is Minkowski, and the radius of curvature of the $AdS$ space provides an effective compactification radius.

Brane world idea # 2: build models where Standard Model gauge and matter fields are restricted to the worldvolume of an intersecting D-brane configuration. We are made of open strings stretched between various pairs of D-branes. Only closed strings (gravitons) can move off-world.
Heterotic string theory on \( CY_3 \)s

The 10D gravity multiplet comes from the NS-NS sector and has the fields \( \{ G_{MN}, B_{MN}, \Phi, \psi^-_M, \lambda^+ \} \) where \( \psi^-_M \) is the gravitino and \( \lambda^- \) is the dilatino. (c.f. Jesse’s Final Project presentation.) Since the heterotic string theory already has a big gauge symmetry in 10D, we also have the vector multiplet \( \{ A_M, \chi^- \} \), where \( \chi^- \) is the gaugino, and we have suppressed the Yang-Mills indices.

The direct product ansatz \( M_{10} = M_4 \times CY_3 \) is not necessarily enough to ensure that the \( \beta \)-function equations are satisfied. For heterotic string theory on a \( CY_3 \), the Bianchi identity like equation for \( B_2 \) yields the important condition

\[
dH_3 = \frac{\alpha'}{4} \left[ \text{Tr} (R \wedge R) - \text{Tr} (F \wedge F) \right] .
\]

This implies that \( H_3 \neq dB_2 \), but rather \( H_3 = dB_2 + \frac{\alpha'}{4} (\Omega_L - \Omega_{YM}) \), where \( \omega_L \) and \( \omega_{YM} \) are the Lorentz and Yang-Mills Chern-Simons terms which play a central role in the analysis of [gauge and gravitational] anomalies.

Since \( \text{Tr} (R \wedge R) \) is nontrivial in cohomology (it is the 2nd Chern character of the tangent bundle), it requires turning on a nontrivial background field strength in order to satisfy \( \text{Tr} (R \wedge R) = \text{Tr} (F \wedge F) \). We ‘embed the spin connection in the gauge group’, using the fact of \( SU(3) \) holonomy group.
Calabi-Yaus

What is a **Calabi-Yau manifold**? It is a Kähler manifold with $n$ complex dimensions and vanishing first Chern class

$$c_1 = \frac{1}{2\pi} [\mathcal{R}] = 0.$$ 

To unpack this definition, let us start with the definition of a **complex structure**. It is a rank $(1,1)$ tensor field $J$ such that $J^2 = -1$ when regarded as an isomorphism on the tangent bundle. In physicist’s terms, $J$ acts like $i$.

A **Kähler manifold** has three mutually compatible structures: a complex structure, a Riemannian structure, and a symplectic structure. You have already met Riemannian manifolds in GR. You have also met symplectic structures in classical mechanics in the part about Hamiltonian dynamics on phase space.

We also need **Chern classes**, which are characteristic classes. These are topological invariants associated to vector bundles on a smooth manifold.

A theorem conjectured by Calabi and proven by Yau says these requirements for Calabi-Yaus imply $SU(n)$ **holonomy**. (For non-compact case, care with BCs at $\infty$ is required to make this fly.) This implies that CYs admit a **covariantly constant spinor**. In turn, this implies compactification on a CY leaves some SUSY preserved. Also, Ricci-flat, in SUGRA approx.
Consider the **exterior derivative** operator $d = dx^\mu \partial_\mu$. This object can be used to act on **differential forms**, which are antisymmetric tensors of rank $p$. For example, in EM we write $F = dA$ where $A = dx^\mu A_\mu$ is the gauge potential and $F$ the field strength tensor. The Bianchi identity is extremely simple: $dF = 0$, and it expresses the fact that $d^2 = 0$. Maxwell equation also becomes v.simple: $\ast d \ast F = j$, where $j$ is the current. For higher $p$-forms, such as NS-NS $B_2$ and R-R $C_{p+1}$, $A_p = \frac{1}{p!} dx^{\mu_1} \wedge \ldots \wedge dx^{\mu_p} A_{\mu_1 \ldots \mu_p}$ where $\wedge$ is the wedge product.

This operator that squares to zero should ring a bell vs. BRST cohomology. Indeed, we can ask: are there differential forms that are closed (killed by $d$) but are not exact (the $d$ of something)? The $p$th **de Rham cohomology** group $H^p(K)$ is defined to be the quotient of closed $p$-forms by exact $p$-forms and it tells us important topological information for compactification.

Next, define the formal adjoint of $d$, known as $d^\dagger$; the Laplacian on forms is then defined by $\Delta = dd^\dagger + d^\dagger d$. This can be used to show that for each de Rham cohomology class on $K$, there is a unique harmonic representative.

Calabi-Yau metric can be written locally in the form $g_{a\bar{b}} = \partial z^a \partial \bar{z}^b \mathcal{K}$ where $\mathcal{K}$ is the **Kähler potential**. Then the Ricci form $\mathcal{R} = dz^a d\bar{z}^b i R_{a\bar{b}}$ is closed because Kähler manifolds have no torsion. This is where the first Chern class arises from.

For more details, see BBS Appendix to §9 starting on p.440, or §14 of BLT.
**Betti numbers** $b_p$ give fundamental topological information about a manifold. $b_p$ is dimension of $p$th de Rham cohomology $H^p(K)$ of manifold $K$. When $K$ has a metric, it counts number of linearly independent harmonic $p$-forms on $K$. When $K$ is Kähler, $b_k = \sum_{p=0}^{k} h^{p,k-p}$ where the $h^{p,q}$ are the Hodge numbers counting the number of harmonic $(p, q)$-forms on $K$. These beasts are very useful for helping figure out the spectrum of dimensionally reduced fields.

A Calabi-Yau is [partially] characterized by its Hodge numbers. The properties of CYs relate $h^{p,0} = h^{n-p,0}$ (use c.c. of $\Omega, g_{ab}$), $h^{p,q} = h^{q,p}$ (c.c.), and $h^{p,q} = h^{n-q,n-p}$ (Poincaré duality). Any compact connected Kähler manifold has $h^{0,0} = 1$ (constant fns). Also, a simply connected manifold has vanishing first homotopy group, and therefore has vanishing first homology group. (Homology and homotopy are both about defining and categorizing holes in a shape but capture different information.) This gives $h^{1,0} = h^{0,1} = 0$. So for $n = 3$, the dimension of interest for us, we only need to specify $h^{1,1}$ and $h^{2,1}$.
Mirror symmetry and the conifold

Calabi-Yaus with $n = 1$ are either $\mathbb{C}$ (non-compact) or $T^2$ (compact). For $n = 2$, you get (a) products of $n = 1$ CYs, for non-compact cases, or for the compact cases either (b) $T^4$ or (c) $K3$. For $n = 3$ the options become much more numerous, with examples including weighted projective spaces. For most CYs the metric is not explicitly known, except in special limits, which makes compactification life more interesting!

CYs are not completely characterized by their Hodge numbers. Indeed, there are pairs of distinct CYs that have the same Hodge numbers and which obey mirror symmetry, which is like a more powerful and complicated version of T-duality. It can be shown at the level of the path integral that the string theories are physically identical for mirror pairs. This involves some pretty math.

Physically, CYs can have deformations. These correspond to changing parameters characterizing their shapes and sizes, described by moduli fields.

Branes wrapped on supersymmetric cycles can become very light under certain conditions analogous to approaching the self-dual radius in circle compactified string theory. It is critical to include these light modes in your low-energy effective action principle or you will miss important nonperturbative effects. A signature example of this is the conifold transition which can, unlike flop transitions, change Hodge numbers. Need NP string theory, not just SUGRA.
Consider probing a big fat $Dp$-brane spacetime with a single ‘test’ $Dp$-brane. Its (kappa symmetric) action in a SUGRA background has two pieces,

$$S_{\text{probe}} = S_{\text{DBI}} + S_{\text{WZ}},$$

which are, to lowest order in derivatives, for the $U(1)$ part,

$$S_{\text{DBI}} = -\frac{1}{g_s (2\pi)^p l_s^{p+1}} \int d^{p+1}\sigma \ e^{-\phi} \sqrt{-\det P (G_{\alpha\beta} + [2\pi F_{\alpha\beta} + B_{\alpha\beta})},$$

$$S_{\text{WZ}} = -\frac{1}{(2\pi)^p l_s^{p+1}} \int P \exp (2\pi F_2 + B_2) \wedge \bigoplus_n C_n .$$

where the $\sigma$ are the worldvolume coordinates and $P$ denotes pullback to the worldvolume of bulk fields.

The brane action encodes both kinetic and potential information, such as which branes can end on other branes. The WZ term, in particular, encodes the fact that $Dp$-branes can carry charge of smaller $D$-branes by having worldvolume field strength $F_2$ (or $B_2$-fields) turned on.

Works for a brane that is topologically $\mathbb{R}^{1,p}$ or wrapped on tori. If the D-brane is wrapped on a manifold which is not flat, extra terms arise in the probe action (e.g. ‘A-roof genus’ for K3). Does not capture non-Abelian physics.
Example: 1 Dp probing N Dps

For our supergravity background exerted by \( N \) Dp-branes, we had

\[
dS^2 = H_p^{-\frac{1}{2}} (-dt^2 + dx^2) + H_p^{\frac{1}{2}} dx^2, \\
e^\Phi = H_p^{\frac{1}{4}(3-p)}, \\
C_{01\ldots p} = g_s^{-1} [1 - H_p^{-1}].
\]

The physics is easiest to interpret in the static gauge, where we fix the worldvolume reparametrisation invariance by setting \( X^\alpha = \sigma^\hat{\alpha}, \alpha = 0, \ldots p \). We also have the \( 9 - p \) transverse scalar fields \( X^i \), which for simplicity we take to be functions of time only, \( X^i = X^i(t), \ i = p + 1 \ldots 9 \). Denote the transverse velocities as \( v^i \equiv \frac{dX^i}{dt} \). Now we can compute the pullback of the metric to the brane.

\[
\mathbb{P}(G_{00}) = (\partial_0 X^\alpha)(\partial_0 X^\beta)G_{\alpha\beta} + (\partial_0 X^i)(\partial_0 X^i)G_{ij} \\
= G_{00} + G_{ij} v^i v^j = -H_p^{-\frac{1}{2}} + H_p^{\frac{1}{2}} \vec{v}^2;
\]

\[
\mathbb{P}(G_{\alpha\beta}) = H_p^{-\frac{1}{2}}.
\]

The last ingredient we need is the determinant of the metric.

\[
\sqrt{-\det \mathbb{P}(G_{\alpha\beta})} = H_p^{-\frac{1}{4}(p+1)} \sqrt{1 - \vec{v}^2 H_p}.
\]
Putting this all together yields

\[ S_{\text{DBI}} + S_{\text{WZ}} = \frac{1}{(2\pi)^{p+1} g_s \ell_s^{p+1}} \int d^{p+1}\sigma \left[ -H_p^{-1} \sqrt{1 - \vec{v}^2} H_p + H_p^{-1} - 1 \right]. \]

From this action we learn that the position-dependent part of the static potential vanishes, as it must for a supersymmetric system such as we have here. The constant piece is of course just the $Dp$-brane tension. In addition, we can expand out this action in powers of the transverse velocity. To lowest order,

\[ S_{\text{probe}} = \frac{1}{(2\pi)^{p+1} g_s \ell_s^{p+1}} \int d^{p+1}\sigma \left[ -1 + \frac{1}{2} \vec{v}^2 + O(\vec{v}^4) \right], \]

so the metric on moduli space, the coefficient of $v^i v^j$, is flat. This is a consequence of having 16 supercharges preserved by the static system.

In SUGRA, SUSY field transformations have a spinorial parameter $\epsilon$. For preserved SUSYs, you find a projection condition for $Dp$-branes. Schematically,

\[(1 + [\text{sgn}(Z)] \Gamma^{01 \cdots p}) \epsilon = 0.\]

So generically, $Dp$-branes break half SUSY. Whether or not $Dp$ and $D(p+4)$ can be in equilibrium is determined by calculation. Find: $Dp \parallel D(p+4)$ is $1/4$ SUSYic.
Open string BCFT

The specific form of the action we presented is valid for the $U(1)$ part of the gauge field only. For a stack of D$p$-branes, generally in an oriented string theory we get $U(N)$ gauge group, not $U(1)$. The story of how the non-Abelian information is encoded in the DBI+WZ action has been worked out in quite some beautiful detail, e.g. in some classic papers of Rob Myers from the 1990s. We have suppressed it here in an attempt to keep the number of details flying around more manageable.

The DBI+WZ action is actually far more than a kappa-symmetric action suitable for D-branes in SUGRA backgrounds. It can be derived in a completely different way as a partial resummation of open string corrections to the $SYM_{p+1}$ action for a stack of $N$ D$p$-branes. This was done in the mid-1980s using worldsheet $\beta$-function techniques and starts with a worldsheet coupling of the form

$$\int_{\partial \Sigma} ds \frac{dX^\mu}{ds} A_\mu(X).$$

There is also an open string tachyon, but the GSO projection gets rid of it too.

*Boundary CFT* methods were exploited nicely in this context. You can find a great deal more detail about how to calculate $\alpha'$ corrections to the lowest-energy $SYM_{p+1}$ Lagrangian using BCFT methods in BLT.
What is an orbifold?

Suppose that $X$ is a smooth manifold with a discrete isometry group $G$. Then we can construct the quotient space $X/G$. A point in that quotient space corresponds to an entire orbit of points in $X$, i.e., a point and all its images under the isometry group. The quotient space has singularities if nontrivial group elements leave points of $X$ invariant (called fixed points).

Locally, the orbifold is indistinguishable from the original manifold.

GR is ill-defined on singular spaces. But strings can actually propagate consistently on manifolds with [spatial*] orbifold singularities! The essential physical reason for this is that strings are extended objects, and so they have significantly softer behaviour at short distance than particles do.

When you mod out by a symmetry, you lose some sectors of the theory, but for orbifolds you also gain back some sectors known as twisted sectors, roughly speaking where the fields only come back to themselves modulo a symmetry transformation. (This is morally similar to what we saw in circle compactified string theory where $KK$ momentum got quantized, losing states, but we also developed a whole new sector of winding modes.)

* If you try to quotient timelike directions, you generically end up with closed timelike curves. These are very bad for your credibility. Interestingly, people have managed to make some sense of some null string theory orbifolds.
The simplest example of a compact orbifold is $S^1/\mathbb{Z}_2$, where the coordinate $x$ is identified with $-x$. This case is relevant to how the $E_8 \times E_8$ heterotic string is obtained from M theory – by compactifying on the interval $S^1/\mathbb{Z}_2$ with end-of-the-world branes carrying the $E_8 \times E_8$ gauge symmetry.

Alternatively, take the complex plane $\mathbb{C}$ and identify $z$ with $-z$. This produces the orbifold $\mathbb{C}/\mathbb{Z}_2$. What does this space look like? The orbifolding identifies the upper half plane’s positive real axis with its negative real axis under a group transformation, and it is a cone. The conical deficit angle is $\pi$. The point $(0,0)$ is a fixed point of the group action.

We could also consider $\mathbb{C}/\mathbb{Z}_N$, where the group is generated by a $2\pi/N$ rotation. This time, the singularity at the origin signifies a deficit angle of $2\pi(N-1)/N$ and it is of $A_N$ type (part of the ADE classification).
Spectra of states for orbifolds

**Untwisted sector states** are those which exist on $X$ and are invariant under the symmetry group $G$. You just take states $\Psi$ such that $g\Psi = \Psi$ for all $g \in G$. If your group is finite, then you can make a $G$-invariant state by starting with any representative $\Psi_0$ and superposing all the images $g\Psi_0$.

**Twisted sector states** arise in the following way. For a closed string propagating on an ordinary manifold, we know that translating $\sigma$ by $2\pi$ brings the embedding map field back to itself. But when you have an orbifold, you only need to produce the same map up to a group transformation:

$$X^\mu(\tau, \sigma + 2\pi) = g X^\mu(\tau, \sigma).$$

For orbifolds there are various distinct twisted sectors labelled by the group element used to make the identification. In more fancy mathy language, they are labelled by the **conjugacy classes** of $G$.

For the $\mathbb{C}/\mathbb{Z}_2$ example, it is clear that twisted sector string states enclose the singular point of the orbifold. In the quantum spectrum, individual twisted sector quantum states are localized at the orbifold singularities that the classical configurations enclose. It is easy to see this approximately for low-lying string states; harder for full-on oscillator content.

Orbifolding enables breaking of some SUSYs of the original manifold.
What is an orientifold?

The one superstring theory we have hardly talked about is Type I, open superstring theory. It can be understood as arising from a projection of Type IIB.

For IIB theory, the two worldsheet superpartners of $X^\mu$ have the same chirality. Worldsheer parity $\Omega : \sigma \rightarrow -\sigma$ is therefore a $\mathbb{Z}_2$ symmetry of the theory, as it exchanges left- and right-moving modes. The bright idea people had was to gauge this worldsheet parity symmetry, and this is what produces Type I from Type IIB.

Recall that T-duality switches the sign of the right-movers only. Then in the T-dual picture, the symmetry transformation above becomes a product of world-sheet parity and a spacetime reflection in directions that were T-dualized.

The closed-string spectrum is obtained by keeping states that are even under $\frac{1}{2}(1 + \Omega)$. The projection condition kills the Kalb-Ramond field and leaves the string metric and dilaton fields preserved. For the gravitini, only the sum of the two survives the projection. Similarly, only one of the IIB dilatini survives, so there are overall $56 + 8 = 64$ massless fermionic d.o.f.

How about the R-R sector? Requiring SUSY implies that there are 64 bosonic d.o.f. A simple light-cone counting shows that $C_0$ and $C_4$ are projected out, leaving only the R-R 2-form $C_2$. This time, the counting is $35 + 1 + 28 = 64$. 
What are $O_p$-planes good for?

One of the main applications orientifold planes have found in string compactifications is to provide sinks for sources of flux. These arise because of tadpole cancellation conditions stating that the total number of sources and sinks must be balanced out to zero overall. Turning on fluxes is one way to help address the perennial problem of *stabilization of moduli*, so it comes up often.

One application this found was to creating string theoretic models of Randall-Sundrum compactifications involving warped product spaces. H.Verlinde showed how if you concentrated D-branes in a Calabi-Yau nearby one another, such as for D3-branes of the AdS/CFT correspondence, you would develop a long $AdS$ throat with a warp factor that scales exponentially in the radial coordinate away from the D3-brane stack.

One may also ask whether $O_p$-planes have other physical roles. They have a very important property other than negative charge (see above): *negative tension*. If you are a half-decent theoretical physicist, the idea of negative-tension objects should jolt you out of your seat, because they will reliably destabilize the vacuum of your theory if they are allowed to fluctuate. But since the $O_p$-planes are actually fixed planes, they do not have a physical interpretation familiar from classical physics. They never fluctuate. Ever.
Flux compactifications

Cycles in e.g. a Calabi-Yau manifold can have fluxes threading through them. What dimension of cycle you need is obtained by looking at how to integrate up your field strength or its Hodge dual:

$$\int_{\gamma_{p+2}} F_{p+2} \quad \text{or} \quad \int_{\gamma_{D-p-2}} \star F_{p+2}.$$ 

Fluxes are actually quantized. If they are sourced by D-branes, then it is clear why fluxes are integers: this corresponds to having an integer number of branes. For manifolds of nontrivial homology, under special conditions, integer quantized fluxes can also be turned on even when there are no brane sources. Flux quantization arises from the generalized Dirac quantization conditions (electric charge $e$ and magnetic charge $g$ obey $e \cdot g \in 2\pi \mathbb{Z}$, via holonomy).

What kinds of fluxes can be turned on, and how they can be mutually compatible, depends sensitively on the type of superstring theory you start with and the compactification you choose. Their options are highly constrained by the ultraviolet physics of the worldsheet.

One of the most difficult tasks in building a relatively realistic string compactification is stabilizing the moduli. Two ubiquitous types of moduli are the dilaton field and the radial modulus describing the overall scale of the CY.
Flux-ology

How do you decide what fluxes to turn on? First of all, if you want low-energy SUSY, you have to ensure that their presence is consistent with the existence of covariantly constant spinors. The fluxes end up appearing in covariant derivatives of spinors, contracted with an antisymmetric combination of gamma matrices. (For example, in IIB compactifications you find that the 3-form field strength needs to be imaginary self-dual to preserve SUSY properly.) Tadpole cancellation conditions also provide an important constraint.

Flux compactifications typically produce a non-trivial scalar potential for the [would-be] moduli fields. A handwaving description of this goes as follows. We may have a modulus field $\phi$ that couples differently to two different fluxes,

$$ S \supset - \int e^{-\phi}|F_1|^2 + e^{+\phi}|F_2|^2. $$

The value of $\phi$ can get dynamically fixed by the ratio of fluxes via the equations of motion.

Very few models have been constructed in which all moduli are stabilized without nonperturbative effects. Nobody knew how to do nonperturbative string theory with any confidence until the second superstring revolution of the mid-1990s.
Inflation in string theory

BBS §10.7 has a nice quick introduction to early universe cosmology and inflation. Unfortunately I have no time to give a quick review here so must refer you there. I can also recommend the gigantic review on string theory modelling of early universe cosmology available from Baumann and McAllister, arxiv:1404.2601 for anyone who has an appetite for more. It will be published soon in proper textbook form by Cambridge University Press.

String theory compactifications have many moving parts. It is possible to obtain inflation, or a phenomenon producing many of the same effects in our universe today, using quite a number of different string theoretic mechanisms. Examples of classes of models that have been studied include the following (borrowed from the B-McA table of contents):

- Inflating with unwarped or warped branes, such as D3-D7.
- Inflating with relativistic branes - Dirac-Born-Infeld style.
- Inflating with axions. Compactified Kalb-Ramond components routinely end up giving rise to an axion field from string theory.
- Inflating with Kähler moduli.
- Inflating with dissipation.

You can also build ekpyrotic universe type models as well, although those suffer from their own controversies.
KKLT and the Landscape

Can we build string models with small positive cosmological constant?

H. Verlinde late 90s: Consider a CY$_3$ with a certain number of O3-planes. Tadpole cancellation allows you to have a limited number of D3-branes living at various points in the CY$_3$. Idea: group enough D3-branes together to make a long AdS$_5$ throat. This introduces a way of understanding developing large hierarchies via exponentially large redshifting, à la Randall-Sundrum.

[K]KLT early 2000s: put in an anti-D3 to break SUSY spontaneously and uplift AdS vacua slightly to deS. Vacua obtained in this way are only metastable. But if their lifetime is extremely long then it does not bother us. The big question is: how controlled is the approximation of adding the anti-D3?

The embarrassing thing about the second superstring revolution is that it eventually yielded the stark realization that there is an extremely large number of possible Standard-Model-like vacua in the theory. It looks very unlikely that our universe would be the only solution to the fundamental equations of string theory. It now seems much more likely that we are a statistical accident.

In this context, the value of the cosmological constant is seen to be a fortunate environmental accident. (Hot tip: believe Susskind, not Smolin!)

String theorists who felt bereft after these revelations have satisfied themselves with investigating the statistical properties of superstring vacua.
Ur potential

Perhaps $10^{1000}$ different minima

Lerche, Lust, Schellekens 1987

Bousso, Polchinski; Susskind; Douglas, Denef, ...