Plan for rest of semester?
Remaining lectures are few! :-
(1) today (string theory & BH entropy)
(2) Fri 03 Dec (extra dimensions & branes?)
(3) Wed 08 Dec. (review lecture &/ question session)

V.I.Q.: (a Very Important Question) ▶ When would you prefer to spend 15–20 minutes on filling out evaluation. Now, the Faculty (of Arts & Science) recommends first part of lecture...
⟶ Your preference = ?

	Fri 03 Dec	Wed 08 Dec	(today?!)
#		✓ ~½-way through	

▶ Please read :-
★ These*, including written comments, are used to help decide whether or not I earn tenure at UofT. Please take care in fulfilling your duty, and if you have criticisms they are usually most effectively phrased (⇒ something will happen! ☺) in terms of constructive criticism...
And if you have nice things to say, don't forget to put pen to paper to express them.
Overall – it's totally your choice what to say, of course! All I ask is that you do it ethically and with care. Cheers!

[* formal evaluations, for Arts & Science]

Kerr Black Hole

[Kerr Fest website]

[Roy Kerr @ University of Canterbury taught me 2nd-year multivariable calculus].

Famous Kerr solution describes rotating BH and is probably the single most important GR metric apart from cosmologies (see GR II)

Stationary. See symmetry under ∂_t and ∂_φ :-

(1.1) $$ds^2 = -\left(1 - \frac{2GMr}{\rho^2}\right) dt^2 - \frac{2GMar\sin^2\theta}{\rho^2}(dt\,d\varphi + d\varphi\,dt) + \frac{\rho^2}{\Delta} dr^2 + \rho^2 d\theta^2 + \frac{\sin^2\theta}{\rho^2}\left[(r^2+a^2)^2 - a^2\sin^2\theta\,\Delta\right] d\varphi^2$$

where

(1.2) $$\Delta = r^2 - 2GMr + a^2 = \Delta(r)$$

and

(1.3) $$\rho^2 = r^2 + a^2\cos^2\theta = \rho^2(r,\theta)$$

Boyer-Lindqvist coordinates

⊛ Angular momentum $J \equiv a \cdot M$

(1.4) ▶ Horizons where $g^{rr}(r_H) = 0$

(1.5) is $r_\pm = GM \pm \sqrt{G^2M^2 - a^2}$ $\Rightarrow$ $|a| \leq GM$ (Cosmic Censorship)

These are actually not Killing horizons for ∂_t. Norm is (C 6.83): $-(\Delta - a^2\sin^2\theta)/\rho^2 > 0$ outside r_+!

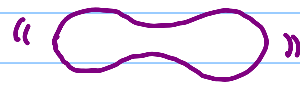
(1.6) Ergosphere : $(r_{erg} - GM)^2 = (G^2M^2 - a^2\cos^2\theta)$

∂_t spacelike inside ergosphere.

Lots of cool stuff to learn - dig into Carroll ☺

So how about string theory: why is it quantum gravity?

Idea #1: World-sheets in space-time.

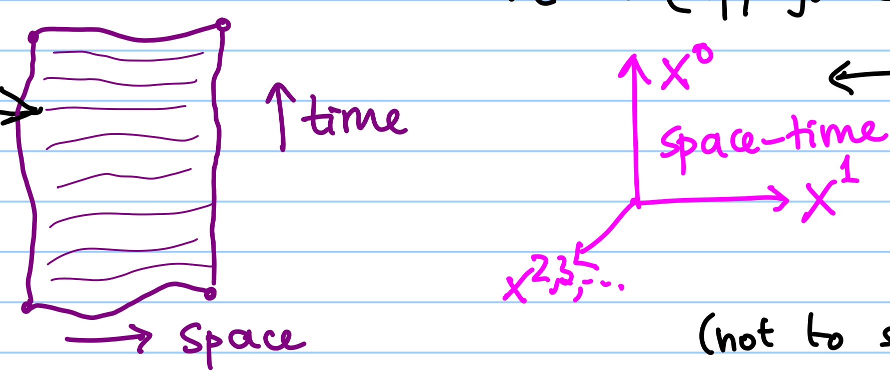
- We know it's possible to have two types of strings:
  - "open", i.e. has ends 
  - "closed", i.e. has no ends ""

- Typically, with any object, a physicist is most interested in knowing how the object moves: "dynamics". It's least complicated, to begin with, to assume that the space-time in which these strings move is flat – like, we're not near a black hole or anything ☺, and we've only got a few strings so they don't cause warping of space-time. OK.

- So we can wonder – when a string moves through space-time, what sort of surface does it sweep out?
- If we watch an open string, it sweeps out a shape like a (ripply) strip:

This is called the worldsheet. ($\mathcal{M}$)

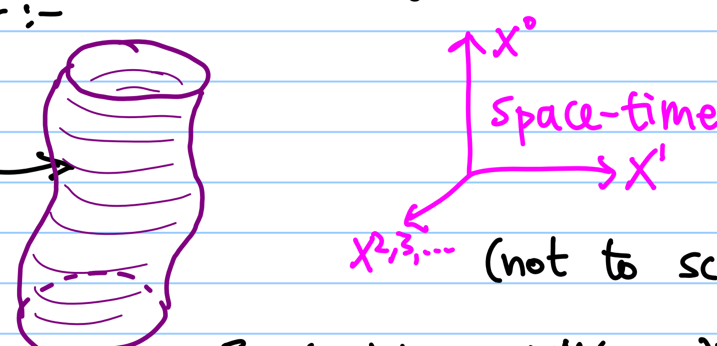
intrinsic geometry



(not to scale)

- What if we have a closed string? It sweeps out a (ripply) cylinder:-

This is called the worldsheet [≠ spacetime!!]



(not to scale)

- $S \propto (\text{tension}) \int_{\mathcal{M}} d\,(\text{area}) \longleftarrow [\text{embedding } X^\mu(\tau, \sigma)]$

Idea #2: Quantum Jitter

- The world is quantum – everything in the universe has a basic jitter even at absolute zero temperature. One way of thinking about this is via the Heisenberg Uncertainty Principle (HUP) which says: (e.g.:) you can't measure time and energy at the same time at infinitely good precision. (Also, there are other pairs of variables afflicted by this basic quantum jitter or uncertainty.) Surprisingly, an electron doesn't even have a definite energy if we measure it over any finite (or zero) time interval! That's kind of surprising, because we're used to saying that objects have definite qualities. But the world is quantum!

- Reason why quantum jitter doesn't bother you in your everyday life is that for big objects – like humans, hockey pucks and dogs – it's so small as to be completely un-noticeable. ☺

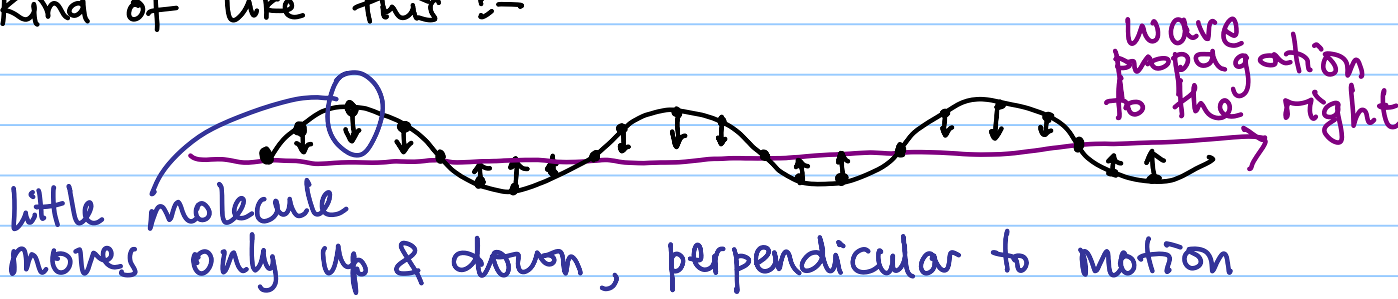
- So... why does quantum jitter affect string physics? Well, strings have ways to wiggle – the technical word for wiggles is "oscillations". And these oscillations are subject to the laws of quantum physics too! We can look separately at each of the different possible oscillation modes ("the oscillators") and ask about their quantum jitter.

- There are two pieces of physics, for string oscillator quantum jitter, that make the story a bit more intricate than you might guess:
  - ⊛ only $(d-2)$ oscillation directions count, when there are d possible directions of space-time;
  - ⊛ the string oscillations can't spread out as much as they want to, and this gives a negative contribution to the mass-squared.

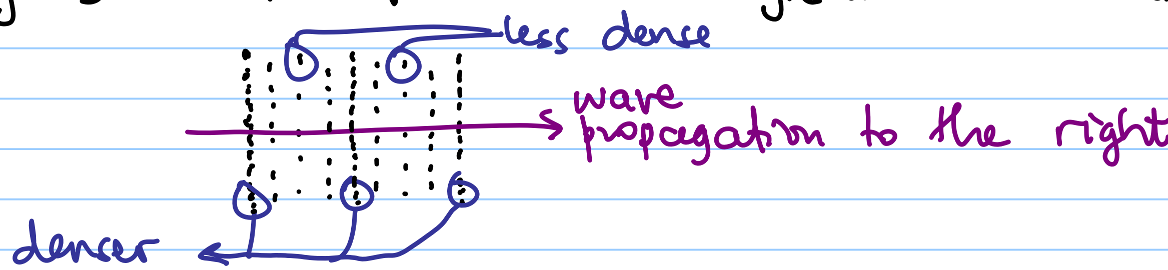
Idea #3 : No longitudinal oscillations of (relativistic) string

Let's think about sound waves. When we send sound through e.g. air or water, there are two qualitatively different types of air-molecule motions that transmit sound: longitudinal and transverse.

Transverse: If we look at a few of the air molecules participating, we see that they move perpendicular to the direction in which the sound wave is propagating, kind of like this :-



Longitudinal Here, the sound wave is propagated through the medium (like air or water) because there can be regions of the medium with greater or smaller density:



For strings, which are [thought to be] the most fundamental things in the universe, there is no medium in which string oscillations propagate! So there are actually no physical longitudinal modes of strings.

Next, let's move to thinking about that basic jitter of every quantum thingy in the universe, even at absolute zero, which is called the zero-point energy.

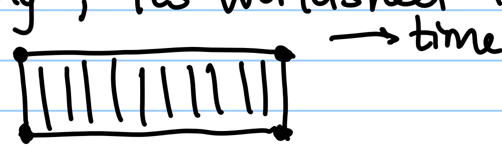
Idea #4 : "Casimir energy"

One of the things about quantum jitter is that everything wants to do the jittering : electrons, photons ; particles ; strings ... everything ! The other relevant thing is that quantum thingys (like electron-particles, or strings) actually are very messy at heart and they like to spread out their ability to jitter all over space.

Moreover, if you arrange for some piece of space to be excluded (somehow) from the quantum jitter options of any thingy, it gives an energy deficit, which is given the name "Casimir energy" (after some old white dead guy called Casimir who discovered the effect ☺). Basically, this energy deficit ΔE is roughly

$$\Delta E = -\begin{pmatrix}\text{the volume of} \\ \text{excluded space}\end{pmatrix} \times \begin{pmatrix}\text{the zero-point energy} \\ \text{density for the thingy}\end{pmatrix}.$$

Now, for an open string, its worldsheet in space-time looks like a strip:



and, since this worldsheet only covers a limited region of space, it has a Casimir deficit. (relativistic)

Similarly, the closed string also has only a restricted region of space covered by its worldsheet :

so it gets a Casimir deficit too. (It turns out to be a slightly bigger deficit.)

Now we can write the mass formula! Using a calculation of the Casimir deficit, our knowledge of the lack of longitudinal oscillations, and quantum physics of relativistic strings gives (da-dah! ☺) :-

Casimir deficit { −4 , closed string ; −1 , open string }

$$\left(\frac{mc^2}{T}\right)^2_{closed} = 2(N-2)$$ $N = 2n \geq 2$ for closed string (n = positive integer)

or

$$\left(\frac{mc^2}{T}\right)^2_{open} = (N-1)$$, $N \geq 1$ for open string (N = positive integer)

In these expressions,
N is the oscillator energy in the string, in units of T,
(mc^2) is the mass of the string. &
T is the tension of the string.

Let's do a couple of examples!

1) Closed: massless modes with $mc^2 = 0$ require $N = 2$.
The two directions in which these two oscillators point signify a spin-two object i.e. the graviton.

2) Open: massless $mc^2 = 0$ requires $N = 1$. The direction in which this single oscillator points signifies a spin-one object, i.e. the photon.

The fact that open & closed strings can give rise to massless gravitons & photons is part of why we call it a Unified Theory. ☺ Also: can prove low-energy dynamics of strings includes GR + other stuff

(*) One more thing. The Casimir deficit story actually also forces us to have, for consistency,
transverse dimensions = 8

and since # dimensions = # transverse + # longitudinal
$= \underbrace{8}_{\text{transverse}} + \underbrace{1 + 1}_{\text{space \& time in string worldsheet}}$

$\Rightarrow$ **# dimensions = 10** ☺ Cool!

> 4

Recent progress

Much! 3 highlights from many :-

- New black holes found in $d>4$ (!)

One is (sorry, can't help it!) BMPV black hole

$$ds_5^{2\,(ext)} = -\left(1-\frac{\mu}{r^2}\right)^2 \left[dt - \frac{\mu\omega \sin^2\theta}{(r^2-\mu)} d\varphi + \frac{\mu\omega\cos^2\theta}{(r^2-\mu)} d\psi\right]^2 + \left(1-\frac{\mu}{r^2}\right)^{-2} dr^2 + r^2(d\theta^2 + \sin^2\theta d\varphi^2 + \cos^2\theta d\psi^2)$$

$J_\varphi = -J_\psi$; mass $\propto \mu$; charge $\propto \mu$; $J_{\varphi,\psi} \propto \omega.\mu$.

- Exact statistical mechanical accounting for S_{BH} via quantum string theory
  (supersymmetric BH and some non-susy BH... even have Hawking radiation & greybody factors!)

⊛ Would you like me to put some weblinks on the lecture notes sector of the course website so you can learn more later as you desire?

? ? ? $(Yes)^3$

- Gravity / gauge holographic dualities.

Connections between gravity & QFT ☺
Learn much both ways.

[String theory PHY2406F - last lecture on this.]