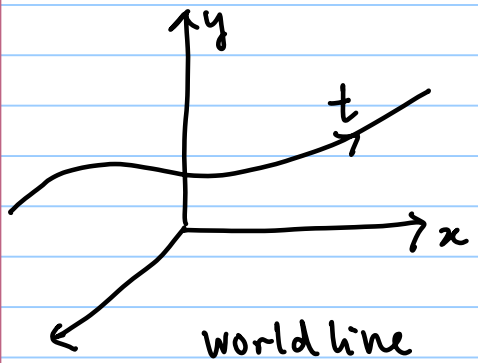
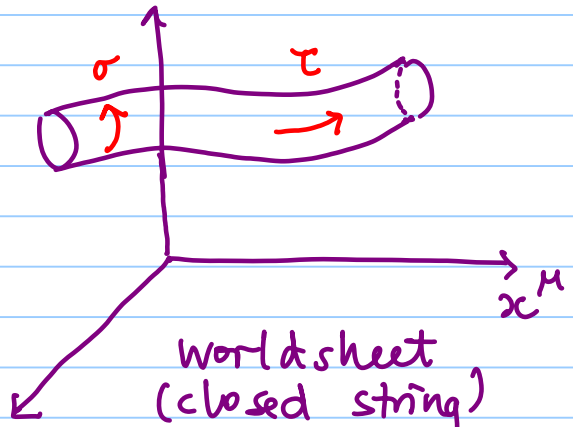
What Is String Theory?

Fundamental idea: replace point particles as basic constituents with strings:



worldline
$x^i(t)$
$\rightarrow x^M(\tau)$
relativistically



worldsheet
(closed string)
described by
$x^M(\tau, \sigma)$

- Think of $x^M(\tau, \sigma)$ as a map from the worldsheet with coördinates (τ, σ) to the ambient spacetime with coörds x^M

▷ Question: how about coörd transformations?

For the particle, it's not necessary to use proper time on the worldline (More soon!)

For the string, there is a much bigger symmetry group of coordinate transformations involving (τ, σ). We will develop the physics of this too.

* Can have both open strings (has endpoints) and closed strings (no ends)

Why Bother?

This complication may seem unnecessary, but it allows unification in a way not possible in particle theory.

Knit together SM + GR

If in flat spacetime, globally (also locally in curved space) symmetry group of {invariances/covariances} of physics

$ISO(1,3) \cong$ rotations + boosts + translations

- rotations: 3 angles (compact)
- boosts: 3 hyperbolic angles (non-compact)
- translations: 4 params (non-compact)

Classify particles by labels (quantum #s) which are invariant under Poincaré: (m^2, s)

- mass: m^2
- spin: $s \in \mathbb{Z}^+ \cup \{0\}$: "bosons"; $s \in \mathbb{Z} + \frac{1}{2}$: "fermions"

Standard Model (SM) of Particle Physics has

Yang-Mills:
- spin-1: interaction-transmitting particles (e.g. photons)
- spin-1/2: matter particles (e.g. electrons)
- spin-0: ? Higgs ← not yet found; thought to be responsible for giving particles mass

General Relativity (GR) has

- spin-2: interaction-transmitting particle (graviton, also not yet seen directly)

interacts with all of the above - including itself.

Riemannian geometry

▷ How to marry these disparate theoretical structures together in a mathematically consistent fashion?

Dimensional analysis

Can unification be achieved?

▷ Physics tool: examine dimensionful & dimensionless parameters in the theory.

E&M: e = charge on electron "$F = \frac{q_1 q_2}{4\pi r^2}$"
$\hbar$ = Planck's constant (QM!)
$d=3+1$ c = speed of light

Dimensionless figure of merit –

$$\alpha_{em} \equiv \frac{e^2}{4\pi\hbar c} \simeq \frac{1}{137}$$ from experiment

$\alpha_{em}^2 \ll 1$ makes perturbation theory possible.

GR: G = Newton's constant "$F = -\frac{G m_1 m_2}{r^2}$"
$\hbar$ (again)
$d=3+1$ c = speed of gravity ← [Einstein assumed same as lightspeed]

No dimensionless combination here;

$$\ell_p = \sqrt{\frac{G\hbar}{c^3}} \sim 10^{-35}\,m \quad \ll \ell_{colliders}$$

$$m_p = \sqrt{\frac{\hbar}{cG}} \sim 10^{-8}\,kg \quad \gg m_{particles(known)}$$

Gravity is weak at our present energy scales.

* Dimensional arguments change in higher dimensions. See Chapter 3 for details.

Questions: Can we solve the dimensional "mismatch"? Can Einstein's theory be generalized to a theory not deeply allergic to QM?

Unification

- String waves (like on a guitar string) oscillate; to a decent approximation small oscillations obey a linear wave eqn = 2nd order PDE.

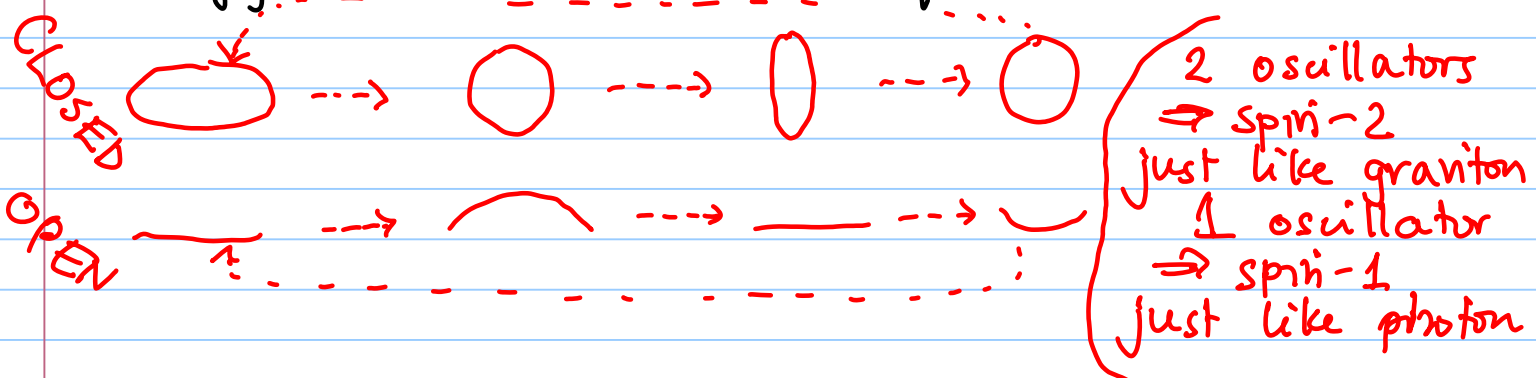
- Symmetry upon going round a closed string $\sigma \to \sigma + 2\pi$ results (c.f. Bloch's theorem in condensed matter physics) in solutions being expressed in terms of Fourier modes [irreps of $U(1)$]
  * Periodic boundary conditions → travelling waves.

- For an open string, boundary conditions are that no momentum leaks off end of string
  ie. * obtain standing waves. (see guitar ☺)

▷ Quantum consistency conditions (we will derive them later on) demand that

Closed-string groundstate has 2 oscillators
Open-string groundstate has 1.

Quantizing a relativistic string is essentially keeping track of all possible SHO degrees of freedom, that satisfy symmetry constraints, given the available energy and other conserved quantum numbers.



This is a rough demonstration of how string theory can possibly envelope both gauge theories and GR.