Quantizing the open string – Part II

Last time, we started on quantizing the open string (using the canonical formalism, in light-front gauge).

We • wrote down the mode expansion for $X^\mu(\tau,\sigma)$

(1.1) $$X^I(\tau,\sigma) = x_0^I + 2\alpha' p^I \tau + i\sqrt{2\alpha'}\sum_{n=1}^{\infty}\left(\frac{1}{n}\alpha_n^I e^{-in\tau} - \frac{1}{n}\alpha_{-n}^I e^{in\tau}\right)\cos(n\sigma)$$

• Had a first glimpse of the Virasoro generators

$$\alpha_n^- = \frac{1}{\sqrt{2\alpha'}\,p^+} L_n^\perp \equiv \frac{1}{2}\sum_{p\in\mathbb{Z}} \alpha_{n-p}^I \alpha_p^I$$

(from LF gauge constraint giving $X^-(X^I\text{'s})$)

(1.2) $$L_0^\perp + a = \alpha' p^I p^I + \sum_{p\in\mathbb{N}} p\, \underbrace{a_{-p}^I a_p^I}_{\#\text{ operator}}$$

so that

(1.3) $$\boxed{\alpha' M^2 = \sum_{n=1}^{\infty} n\, a_{-n}^I a_n^I + a}$$

$$a = -\frac{(D-2)}{24}$$

Virasoro algebra

(1.4) Define $$\boxed{L_n^\perp = \sum_{p\in\mathbb{Z}} \frac{1}{2}\alpha_p^I \alpha_{n-p}^I}$$

Last time we discussed the fact that

• α_p^I only fail to commute when have same index

i.e. normal-ordering of α_p^I operators matters only for $L_0^\perp$. But it's crucial, because $L_0^\perp$ is the Hamiltonian on the worldsheet in light-front gauge (see e.g. Z (12.16).) !

Hermiticity

Since $(\alpha_p^I)^\dagger = \alpha_{-p}^I$, $(\alpha_n^-)^\dagger = \alpha_{-n}^-$, so that

(2.1) $$\boxed{(L_n^\perp)^\dagger = L_{-n}^\perp}$$

Previously, we learned that

$$[\alpha_m^I, \alpha_n^J] = m\,\delta_{m+n,0}\,\eta^{IJ}$$

From this, and the definition (1.4), it follows straightforwardly that

(2.2) $$\boxed{[L_m^\perp, \alpha_n^J] = -n\,\alpha_{m+n}^J}$$

Notice that

- the mode # on the LHS of (2.2) is the same as the mode # on the right (✓)
- the spatial index structure is also preserved (✓)

One more application of commutators and the definition of $L_n^\perp$ gives

$$[L_m^\perp, L_n^\perp] = (m-n)\,L_{m+n}^\perp, \quad m+n \neq 0$$

Finding the result for $m+n=0$ is significantly more work but again can be done; this time, you end up needing (again) to use that subtraction procedure on the zero-point energies sum that we saw last time. Then, finally (see Zwiebach pp 222-226)

(2.3) ✱ ✱ $$\boxed{[L_m^\perp, L_n^\perp] = (m-n)\,L_{m+n}^\perp + \frac{(D-2)}{12}(m^3-m)\,\delta_{m+n,0}}$$

Notice in particular how the quadratic story for α_n^- in terms of α_n^I leads to a very different set of commutation relations for the α_n^- ($L_n^\perp$) as compared to the CCR's for the α_n^I !

▷ Go through the derivation for yourself. !

The Virasoro algebra is very important for string theory.

Let us see what they do physically for our open string. (Sneak peek: $L_m^\perp$ will actually generate for us reparametrizations of the worldsheet!)

Using the mode expansion (1.1), we can figure the commutator

$$[L_m^\perp, X^I(\tau,\sigma)] = [L_m^\perp, x_0^I] + i\sqrt{2\alpha'}\sum_{n\neq 0}\frac{1}{n}\cos(n\sigma)\,e^{-in\tau}[L_m^\perp, \alpha_n^I]$$

$$= -i\sqrt{2\alpha'}\,\alpha_m^I - i\sqrt{2\alpha'}\sum_{n\neq 0}\cos(n\sigma)\,e^{-in\tau}\alpha_{m+n}^I$$

where we used $\alpha_0^I = \sqrt{2\alpha'}\,p^I$, $[x_0^I, p^J] = i\eta^{IJ}$ and eqn (2.2)

so

$$[L_m^\perp, X^I(\tau,\sigma)] = -i\sqrt{2\alpha'}\sum_{n\in\mathbb{Z}}\cos(n\sigma)\,e^{-in\tau}\alpha_{m+n}^I$$

$$= -\frac{i}{2}\sqrt{2\alpha'}\sum_{n\in\mathbb{Z}}\left[e^{-in(\tau+\sigma)} + e^{-in(\tau-\sigma)}\right]\alpha_{m+n}^I$$

$$= -\frac{i}{2}\sqrt{2\alpha'}\sum_{n\in\mathbb{Z}} e^{-i(n-m)(\tau+\sigma)}\,\alpha_n^I - \frac{i}{2}\sqrt{2\alpha'}\sum_{n\in\mathbb{Z}} e^{-i(n-m)(\tau-\sigma)}\,\alpha_n^I$$

Compare this to derivatives of string coords $X^I(\tau,\sigma)$

$$\Rightarrow [L_m^\perp, X^I(\tau,\sigma)] = -\frac{i}{2}e^{im(\tau+\sigma)}\sqrt{2\alpha'}\sum_{n\in\mathbb{Z}} e^{-in(\tau+\sigma)}\alpha_n^I - \frac{i}{2}e^{im(\tau-\sigma)}\sqrt{2\alpha'}\sum_{n\in\mathbb{Z}} e^{-in(\tau-\sigma)}\alpha_n^I$$

(sum of L- and R- moving pieces)

c.f.

$$\dot{X}^I \pm X^{I\prime} = \sqrt{2\alpha'}\sum_{n\in\mathbb{Z}}\alpha_n^I\, e^{-in(\tau\pm\sigma)}$$

So $$[L_m^\perp, X^I(\tau,\sigma)] = -\frac{i}{2}e^{im(\tau+\sigma)}(\dot{X}^I + X^{I\prime}) - \frac{i}{2}e^{im(\tau-\sigma)}(\dot{X}^I - X^{I\prime})$$

(4.1) i.e. $$\boxed{\left[L_m^{\perp}, X^I(\tau,\sigma)\right] = -i e^{im\tau}\cos(m\sigma)\,\dot{X}^I(\tau,\sigma) + e^{im\tau}\sin(m\sigma)\,X^{I\prime}(\tau,\sigma)}$$

If we were to reparametrize

$$\tau \to \tau + \epsilon\left(-i e^{im\tau}\cos(m\sigma)\right) \equiv \tau + \epsilon\,\xi^{\tau}_m$$

$$\sigma \to \sigma + \epsilon\left(e^{im\tau}\sin(m\sigma)\right) \equiv \tau + \epsilon\,\xi^{\sigma}_m$$

then $$X^I(\tau,\sigma) \to X^I(\tau + \epsilon\,\xi^{\tau}_m, \sigma + \epsilon\,\xi^{\sigma}_m)$$

$$\cong X^I + \epsilon\,\xi^{\tau}_m\,\dot{X}^I + \epsilon\,\xi^{\sigma}_m\,X^{I\prime}$$

$$= X^I + \left[L_m^{\perp}, X^I\right]$$

i.e.

(4.2) $$\boxed{\delta X^I(\tau,\sigma) = \left[L_m^{\perp}, X^I(\tau,\sigma)\right]} \qquad \text{(Virasoro)}$$

(4.3) where $$\delta\tau = \epsilon\,\xi^{\tau}_m = \epsilon\left(-i e^{im\tau}\cos(m\sigma)\right)$$
$$\delta\sigma = \epsilon\,\xi^{\sigma}_m = \epsilon\left(e^{im\tau}\sin(m\sigma)\right)$$

So indeed <u>the Virasoro generators perform worldsheet reparametrizations.</u>

One special application of this is to take $m=0$ so that $\delta\tau = \epsilon \cdot -i$, $\delta\sigma = 0$.

Then eqn (4.2) says that

(4.4) $$\delta X^I = \left[L_0^{\perp}, X^I\right] = -i\dot{X}^I$$

This is Heisenberg equation for time evolution (in QM.)

So indeed $L_0^{\perp}$ <u>is</u> the worldsheet Hamiltonian. ☺

Lorentz algebra

Previously we wrote down the conserved world-sheet currents and hence charges

(5.1) $$M_{\mu\nu} = \int_0^\pi d\sigma \, (X_\mu P^\tau_\nu - X_\nu P^\tau_\mu)$$

Classically

which, in our parametrization, just gives

(5.2) $$M_{\mu\nu} = \int_0^\pi d\sigma \, (X^\mu \dot{X}^\nu - X^\nu \dot{X}^\mu)$$

We really, really, really want to know if there are any delicacies at all involved in defining these chappies as quantum operators!

Using the mode expansions (1.1) and CCR's for α^I_n, gives

(5.3) $$\boxed{M^{\mu\nu}_{(cl)} = x_0^\mu p^\nu - x_0^\nu p^\mu - i \sum_{n=1}^{\infty} \frac{1}{n} (\alpha^\mu_{-n} \alpha^\nu_n - \alpha^\nu_{-n} \alpha^\mu_n)}$$

This is the classical expression in terms of modes. Note that, as required for conserved charges, there is no τ dependence. (The σ integration and orthogonality of Fourier modes was also useful.)

Big Question: is (5.3) valid in Quantum Theory?

Let's see if we can make sense of these dudes.

⊛ Most delicate will be the M^{-I}; in LF gauge anything involving a $-$ index is always more complicated or subtle (because we found X^- in terms of "$(X^I)^2$" using the constraints and the LF gauge condition!)

Let us postulate that (5.3) is OK for $\mu = I, \nu = J$ but that M^{-J} is more subtle.

Try
(6.1) $$\boxed{M^{-I} = x_0^- p^I - \frac{1}{2}\left(x_0^I p^- + p^- x_0^I\right) - i\sum_{n=1}^{\infty}\frac{1}{n}\left(\alpha_{-n}^- \alpha_n^I - \alpha_{-n}^I \alpha_n^-\right)}$$ 6

don't commute

Then

(6.2) $(M^{-I})^\dagger = M^{-I}$ because $(\alpha_n^I)^\dagger = \alpha_{-n}^I$ & $(\alpha_n^-)^\dagger = \alpha_{-n}^-$.

$\Rightarrow$ Hermitean.

Now. What about normal-ordering of operators?

The α_n^- are normal-ordered $\because$ they are Virasoro$^\perp$'s.

So $$M^{-I} = x_0^- p^I - \frac{1}{2}\left(x_0^I \frac{(L_0^\perp + a)}{2\alpha' p^+} - \frac{(L_0^\perp + a)}{2\alpha' p^+} x_0^I\right) - \frac{i}{\sqrt{2\alpha'}\, p^+}\sum_{n=1}^{\infty}\frac{1}{n}\left(L_{-n}^\perp \alpha_n^I - \alpha_{-n}^I L_n^\perp\right)$$

Finding the $[M^{-I}, M^{-J}]$ is long, it involves D and a, but it is do-able.

I recommend this as an exercise.

The central point about this commutator is

(6.3) $$[M^{-I}, M^{-J}] = \frac{1}{\alpha'(p^+)^2}\sum_{m=1}^{\infty}\left(\alpha_{-m}^I \alpha_m^J - \alpha_{-m}^J \alpha_m^I\right) \times$$
$$\times\left\{ m\left[1 - \frac{(D-2)}{24}\right] + \frac{1}{m}\left[\frac{(D-2)}{24} + a\right]\right\}$$

where Zwiebach uses ζ-functions when needed and our previously figured CCR's (LONG calculation)

$\hookrightarrow$ have a go?

Closure of the Lorentz algebra would imply, in this case,

$$[M_{\mu\nu}, M_{\alpha\beta}] = i M_{\mu\alpha}\eta_{\nu\beta} - M_{\mu\beta}\eta_{\nu\alpha} - i M_{\nu\alpha}\eta_{\mu\beta} + i M_{\nu\beta}\eta_{\mu\alpha}$$

but $\eta_{--} = 0$ and $M_{--} = 0$ (antisymmetry) so we require

(6.4) $$[M^{I-}, M^{J-}] = 0$$

This requires, $\forall m$, $1 - \frac{(D-2)}{24} = 0$ and $\frac{(D-2)}{24} + a = 0$

Solved by $\boxed{D=26}$ and $\boxed{a=-1}$

Critical dimension — Zero-point energy

24 Transverse Coordinates

This is a FRIGGIN' HUGE RESULT!

By insisting that Lorentz symmetry generators act properly as quantum operators
- Hermitean
- obey Lorentz algebra

we have just derived the critical dimension ($D=26$) of bosonic string theory. WOO HOO!

In superstring theory, which we will touch upon later, the critical dimension becomes 10. (The reason is that fermions also contribute zero-point energy...)

Constructing open-string state space

Introduce ground states $|p^+, \vec{p}_T\rangle$

(7.1) and demand $a_n^I \, |p^+, \vec{p}_T\rangle = 0 \quad , \; n \geq 1$

Act on these babies with $\{(a_n^J)^\dagger \, ; J, n\}$ to make states

(7.2) $$|\lambda\rangle = \prod_{n=1}^{\infty} \prod_{I=2}^{25} \left(a_n^I{}^\dagger\right)^{\underbrace{\lambda_{n,I}}_{\#\text{ times the } (a_n^I)^\dagger \text{ appears}}} |p^+, \vec{p}_T\rangle$$

A general state will be a superposition of these basis states $|\lambda\rangle$.

What does a state with a particular $\{\lambda_{n,I}\}$ look like?

$$\alpha' M^2 = N^\perp - 1$$

where $N^\perp \equiv \sum_{n=1}^{\infty} n (a_n^I)^\dagger a_n^I$ $\leftarrow$ (from $L_0^\perp = \alpha' p^I p^I + N^\perp$)

(8.1)
$$\begin{cases} \text{and in fact } \alpha' M^2 = N^\perp - 1 \\ [N^\perp, a_m^I] = -m\, a_m^I \quad \text{and} \quad [N^\perp, (a_m^I)^\dagger] = +m (a_m^I)^\dagger \\ N^\perp \,|p^+, \vec{p}_T\rangle = 0 \end{cases}$$

Example:

$$N^\perp (a_3^J)^\dagger (a_2^I)^\dagger \,|p^+, \vec{p}_T\rangle$$

$$= \left([N^\perp, (a_3^J)^\dagger] + (a_3^J)^\dagger N^\perp\right)(a_2^I)^\dagger \,|p^+, \vec{p}_T\rangle$$

$$= 3 (a_3^J)^\dagger (a_2^I)^\dagger \,|p^+, \vec{p}_T\rangle + 2 (a_3^J)^\dagger (a_2^I)^\dagger \,|p^+, \vec{p}_T\rangle + 0 + 0$$

$$= 5 (a_3^J)^\dagger (a_2^I)^\dagger \,|p^+, \vec{p}_T\rangle$$ i.e. <u>5</u> units of stringy excitation

More generally,

$$N^\perp |\lambda\rangle = N_\lambda^\perp |\lambda\rangle \quad \text{where}$$

(8.2) $$N_\lambda^\perp = \sum_{n=1}^{\infty} \sum_{I=2}^{25} n\, \lambda_{n,I}$$

Normalization of groundstate:-

(8.3) $$\langle p'^+ \vec{p}_T{}' \,|\, p^+, \vec{p}_T\rangle = \delta(p'^+ - p^+, \vec{p}_T{}' - \vec{p}_T)$$

Bras dual to kets $|\lambda\rangle$ are

$$\langle\lambda| = \langle p^+, \vec{p}_T| \sum_{n=1}^{\infty} \sum_{I=2}^{25} (a_n^I)^{\lambda_{n,I}}$$

For the inner product,

(8.4) $$\langle \chi|\lambda\rangle = \langle p'^+, \vec{p}_T{}' \,|\, a_1 (a_1^J)^\dagger \,|p^+, \vec{p}_T\rangle$$

$$= \delta^{IJ}\, \delta(p^{+\prime} - p^+)\, \delta(\vec{p}_T{}' - \vec{p}_T)$$

We can now use these results to do some well-known results and REALLY read them!

So. a being -1 $\Rightarrow$ tachyon @ low-E mode.
Next step up is to compute for interesting others.

With $(a_1^I)^\dagger |p^+, \vec{p}_T\rangle$ one has $M^2 (a_1^I)^\dagger |p^+, \vec{p}_T\rangle$

(9.1) $M^2 = 0$ state : $\sum_{I=2}^{25} \xi_I (a_1^I)^\dagger \, |p^+, \vec{p}_T\rangle$

c.f. Maxwell theory, in which

(9.2) $(a_1^I)^\dagger |p^+, \vec{p}_T\rangle \longleftrightarrow a^I_{p^+, \vec{p}_T} |\Omega\rangle$

$\Rightarrow$ Quantum open string massless states are the photon.

The astonishment here is that we get a $m^2=0, s=1$ field out of the second lowest mode.

The above states have $N^\perp = 1$. Trying for $N^\perp = 2$:-
Built via vacuum plus acting with either one oscillator of momentum too : $(\alpha_1^I)^\dagger (\alpha_1^J)^\dagger \, |p^+; \vec{p}_T\rangle$

String field theory works out (computer. mainly)

Modern techniques enabled more.

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Before starting closed string story, recall that

$$n \cdot X = \alpha' (n \cdot p) \tau$$

$$(n \cdot p) \sigma = 2\pi \int_0^\sigma d\tilde{\sigma} \, n \cdot \mathcal{P}^\tau (\tau, \tilde{\sigma}) \qquad \sigma \in [0, 2\pi]$$

Familiar kinematics

$$(\dot{X}^I \pm X^{I\prime})^2 = 0$$

$$\mathcal{P}^{\tau\mu} = \frac{1}{2\pi\alpha'} \dot{X}^\mu , \quad \mathcal{P}^{\sigma\mu} = \frac{-1}{2\pi\alpha'} X^{\mu\prime}$$

and $(\partial_\tau^2 - \partial_\sigma^2) X^\mu(\tau, \sigma) = 0$

Closed String mode expansion

(10.1) $$\boxed{X^\mu(\tau,\sigma) = X_L^\mu(\tau+\sigma) + X_R^\mu(\tau-\sigma)}$$

(10.2) Periodicity : $\boxed{X^\mu(\tau,\sigma) = X^\mu(\tau,\sigma+2\pi)}$ — only for topologically trivial spacetime

Define $u = \tau - \sigma$
$v = \tau + \sigma$

then

(10.3) $$X_L^{\mu\prime}(u) = \sqrt{\frac{\alpha'}{2}} \sum_{n\in\mathbb{Z}} \bar{\alpha}_n^\mu e^{-inu}$$

(10.4) $$X_R^{\mu\prime}(v) = \sqrt{\frac{\alpha'}{2}} \sum_{n\in\mathbb{Z}} \alpha_n^\mu e^{-inv}$$

and integrating P's gives

$$X^\mu(\tau,\sigma) = x_0^\mu + \sqrt{2\alpha'}\,\alpha_0^\mu \tau + i\sqrt{\frac{\alpha'}{2}} \sum_{n\neq 0} \frac{e^{-in\tau}}{n}\left(\alpha_n^\mu e^{+in\sigma} + \bar{\alpha}_n^\mu e^{-in\sigma}\right)$$

Then

$$(\dot{X} + X')^\mu = 2X_L^{\mu\prime}(\tau+\sigma) = \sqrt{2\alpha'} \sum_{m\in\mathbb{Z}} \bar{\alpha}_n^\mu e^{-in(\tau+\sigma)}$$

$$(\dot{X} - X')^\mu = 2X_R^{\mu\prime}(\tau-\sigma) = \sqrt{2\alpha'} \sum_{m\in\mathbb{Z}} \alpha_n^\mu e^{-in(\tau-\sigma)}$$

This gives, in very similar way as open string,

$$\boxed{\begin{aligned} [\alpha_m^I, \alpha_n^J] &= m\,\delta_{m+n,0}\,\eta^{IJ} \\ [\bar{\alpha}_m^I, \bar{\alpha}_n^J] &= m\,\delta_{m+n,0}\,\eta^{IJ} \\ [\alpha_m^I, \bar{\alpha}_n^J] &= 0 \end{aligned}}$$

Note the indep. L & R CCR's.

Also, see $[x_0^I, p^J] = i\eta^{IJ}$

(11.1) Then
$$L_n^\perp = \frac{1}{2}\sum_{p\in\mathbb{Z}} \alpha_p^I \alpha_{n-p}^I$$
$$\bar{L}_n^\perp = \frac{1}{2}\sum_{p\in\mathbb{Z}} \bar{\alpha}_p^I \bar{\alpha}_{n-p}^I$$

whence

(11.2)
$$\dot{X}^- + X^{-\prime} = \sqrt{2\alpha'}\sum_{n\in\mathbb{Z}} \bar{\alpha}_n^- e^{-in(\tau+\sigma)}$$
$$\dot{X}^- - X^{-\prime} = \sqrt{2\alpha'}\sum_{n\in\mathbb{Z}} \alpha_n^- e^{-in(\tau-\sigma)}$$

Constraint

(11.3) $\boxed{\bar{L}_0^\perp = L_0^\perp}$ = Constraint on physical states

Define

(11.4) $$\bar{L}_0^\perp = \frac{\alpha'}{4} p^I p^I + \bar{N}^\perp \quad , \quad L_0^\perp = \frac{\alpha'}{4} p^I p^I + N^\perp$$

(11.5) Again, find $\boxed{D=26}$ from Lorentz operator alg.
← would have been worrying if had differed from open!

Then for constants in the normal-ordering,

(11.6) $$\sqrt{2\alpha'}\,\bar{\alpha}_0 = \frac{2}{p^+}(\bar{L}_0^\perp - 1) \quad , \quad \sqrt{2\alpha'}\,\alpha_0 = \frac{2}{p^+}(L_0^\perp - 1)$$

in exact analogy to prior open-string case.

So

(11.7) $$L_0^\perp = \bar{L}_0^\perp \Rightarrow \boxed{N^\perp = \bar{N}^\perp}$$

while we find the mass-squared as

(11.8) $$\boxed{\alpha' M^2 = 2(N^\perp + \bar{N}^\perp - 2)}$$

with

(11.9) $$\boxed{H = p^+ p^- \alpha' = \bar{L}_0^\perp + L_0^\perp - 2}$$

Virasoro algebra

$$[\bar{L}_m^\perp, \bar{\alpha}_n^J] = -n\bar{\alpha}_{m+n}^J \quad ; \quad [L_m^\perp, \alpha_n^J] = -n\alpha_{m+n}^J \quad ;$$
$$[\bar{L}_m^\perp, \alpha_n^J] = 0 \quad ; \quad [L_m^\perp, \bar{\alpha}_n^J] = 0 \quad .$$

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(12.1) $[\bar{L}^\perp_m, x_0^J] = -\frac{i}{2}\sqrt{2\alpha'}\,\bar{\alpha}^J_m$; $[L^\perp_m, x_0^J] = -\frac{i}{2}\sqrt{2\alpha'}\,\alpha^J_m$.

Then (p. 256)

(12.2)
$$\begin{cases} [\bar{L}^\perp_0, X^I(\tau,\sigma)] = -\frac{1}{2}i\,(\dot{X}^I + X'^I) \\ [L^\perp_0, X^I(\tau,\sigma)] = -\frac{1}{2}i\,(\dot{X}^I - X'^I) \end{cases}$$

See the two open-string structures hiding? ☺

(12.3)
$$\Rightarrow \boxed{\begin{aligned} [L^\perp_0 + \bar{L}^\perp_0, X^I(\tau,\sigma)] &= -i\frac{\partial X^I}{\partial\tau} \\ [L^\perp_0 - \bar{L}^\perp_0, X^I(\tau,\sigma)] &= +i\frac{\partial X^I}{\partial\sigma} \end{aligned}}$$

So $(L^\perp_0 - \bar{L}^\perp_0)$ generates translations along σ

(12.4) Writing $\boxed{P \equiv L^\perp_0 - \bar{L}^\perp_0}$, $X^I(\tau,\sigma+\sigma_0) = e^{-i\sigma_0 P} X^I(\tau,\sigma) e^{i\sigma_0 P}$.

P annihilates all closed string states $|\Psi\rangle$:-

(12.5)
$$\boxed{e^{-i\sigma_0 P}|\Psi\rangle = |\Psi\rangle}$$

World-sheet momentum

(not spacetime!)

Closed String State Space

(12.6)
$$\boxed{|\lambda,\bar{\lambda}\rangle = \left[\prod_{n=1}^{\infty}\prod_{I=2}^{25}\left((a_n^I)^\dagger\right)^{\lambda_{n,I}}\right] \times \left[\prod_{n=1}^{\infty}\prod_{J=2}^{25}\left((\bar{a}_n^J)^\dagger\right)^{\bar{\lambda}_{m,J}}\right]|p^+, \vec{p}_T\rangle}$$

$$\boxed{N^\perp = \sum_{n=1}^{\infty}\sum_{I=2}^{25} n\lambda_{n,I}} \; ; \; \boxed{\bar{N}^\perp = \sum_{n=1}^{\infty}\sum_{J=2}^{25} m\bar{\lambda}_{m,J}}$$

$$\boxed{N^\perp = \bar{N}^\perp}$$

$$\boxed{\alpha' m^2 = 2(N^\perp + \bar{N}^\perp - 2)}$$

Work out first few babies!

$\alpha' m^2$
20
18 ×
16
12 ×
8
4 ×
0
−4 ×
GSO proj SST

Closed-string states

$N^\perp = \bar{N}^\perp \Rightarrow \alpha' m^2 = 2(2N-2) = 4(N-1)$

Lowest $N=0 \Rightarrow \boxed{\alpha' m_T^2 = -4}$ (c.f. −1 for O.S.)

tachyon

Massless $\boxed{\alpha' m^2 = 0}$

$\Rightarrow N^\perp = 1,\ \bar{N}^\perp = 1 \Rightarrow$ 2 oscillators

Can arrange two spatial indices into 3 Lorentz reps

$\Rightarrow$ { traceless / antisymmetric tensor / traceless symmetric tensor

Φ
$B_{(2)}$
$g_{\mu\nu}$

We will develop the physics of these 3 soon.

Dilaton: physical meaning

In QED, $\frac{e^2}{4\pi\hbar c} \equiv \alpha_{em}$ is dimensionless.

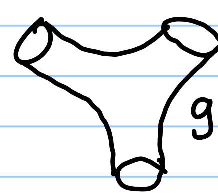
What is the analog in ST?

First, we will have to face dimensions, since

$$16\pi G_D \equiv (2\pi)^{D-3} \ell_p^{D-2}$$ { Newton constant in terms of Planck length

$$16\pi G_{10} = (2\pi)^7 g_s^2 \ell_s^8$$

g_s ← string coupling



$$G_{10-n} = \frac{G_{10}}{V_n} \leftarrow \text{compact volume}$$

$\Rightarrow G \propto g_s^2$ (if $V_n = \mathcal{O}(\ell_s^n)$)

also

$g_s \propto g_{open}^2$

$$\boxed{g_s = \langle e^{\Phi} \rangle}$$