ST 12 NOV

Note Title 12/11/2004

LOW-ENERGY LIMIT OF SUPER STRING THEORY

Nambu-Goto action

(1.1) $$S_{NG} = \frac{1}{4\pi\alpha'}\int d^2\sigma \sqrt{-\det\left(\frac{\partial X^\mu}{\partial\sigma^\alpha}\frac{\partial X^\nu}{\partial\sigma^\beta}\eta_{\mu\nu}\right)}$$

not easy to quantize; especially with Feynman path-integral approach.

Equivalent classically to <u>Polyakov action</u>

(1.2) $$S_p = -\frac{T}{2}\int d^2\sigma \sqrt{-\gamma}\, \gamma^{ab}\partial_a X^\mu \partial_b X^\nu \eta_{\mu\nu}$$
$$+\lambda\left(\frac{1}{4\pi}\int_M d^2\sigma\sqrt{-\gamma}\,{}^{(2)}R + \frac{1}{2\pi}\int_{\partial M} ds\, K\right)$$

<u>N.B.</u>:
- gravity in 2d gets no kinetic term (not dynamical)
- no cosmological constant - not conformally inv't.

Consider piece in path integral

(1.3) $$Z = \int \mathcal{D}\{X, \text{etc}\} \exp(-S_E)$$ like partition fn

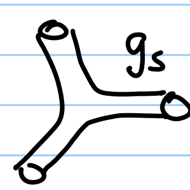
Now, piece in ${}^{(2)}R$ is topological & → Euler #

(1.4) $\Rightarrow$ $$Z \propto \exp(-\lambda[2-2h-b-c])$$

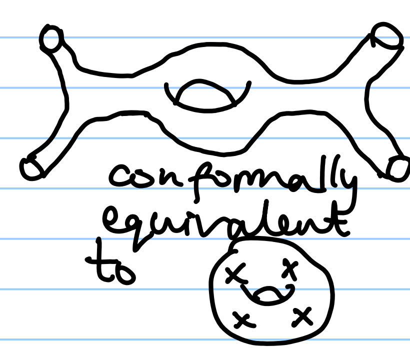
where $\left\{\begin{array}{l} h = \#\text{ handles} \\ b = \#\text{ boundaries} \\ c = \#\text{ crosscaps}\end{array}\right\}$ $g_s = e^{-\lambda}$

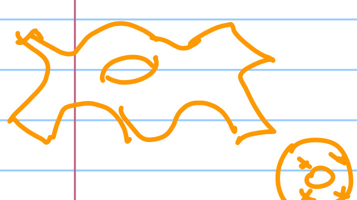
$$\boxed{\langle e^{\Phi}\rangle \equiv g_s}$$

Coupling strength for 3-point closed string vertex.



Since # boundaries comes in with coeff half that of # handles in χ, Euler #, find





So split off constant piece into g_c &

(2.1) $$\boxed{S_{poly} = -\frac{T}{2}\int d^2\sigma \sqrt{-\gamma}\, \gamma^{ab}\partial_a X^\mu \partial_b X^\nu \eta_{\mu\nu}}$$

Then this can be more easily quantized, particularly in CFT where keep covariance at price of Fadeev-Popov ghosts to do bookkeeping of gauge symmetry.

Background fields

Saw massless states of closed string included

(2.2) $$\begin{cases} g_{\mu\nu} & \text{symmetric traceless 2-index tensor} \\ B_{[2]} & \text{antisymmetric 2-index tensor} \\ \Phi & \text{trace scalar} \end{cases}$$

Let's turn on these dudes!

(2.3) $$\boxed{\begin{aligned} \Rightarrow S_{\sigma\text{-model}} = -\frac{1}{4\pi\alpha'}\int d^2\sigma \Big\{ & \sqrt{-\gamma}\, \gamma^{ab}\partial_a X^\mu \partial_b X^\nu G_{\mu\nu}(X) \\ & + \epsilon^{ab}\partial_a X^\mu \partial_b X^\nu B_{\mu\nu}(X)\Big\} \\ + \frac{1}{4\pi}\int d^2\sigma \sqrt{-\gamma}\, {}^{(2)}R\, \Phi(X) & + (\text{higher order in } \alpha') \end{aligned}}$$ (*)

Note that dilaton piece comes at order α' by comparison with g & B terms.

Action (2.3) is only one possible consistent with symmetry (e.g. diffeo invariance) on string worldsheet and in bulk.

(*) Note that $G_{\mu\nu}(X)$, $B_{\mu\nu}(X)$, $\Phi(X)$ are not (explicitly) functions of $(\tau,\sigma) = \{\sigma^a\}$! They are therefore to be thought of as coupling functions of the theory like $1/\alpha_{EM} = e^2/(\hbar c)$ of QED.

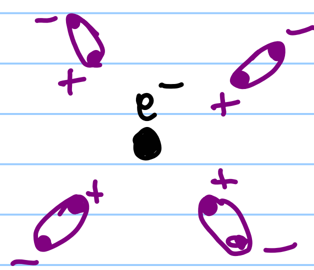
- By analogy with E&M these coupling functions run with energy. But S_{poly} is conformal; demand in quantum theory too! Compute running of couplings.

β-functions

- In any theory the coupling 'constants' depend on energy.
  Quantum, relativistic phenomenon.
  $\hookrightarrow \hbar$ (virtual particles) available $\quad \hookrightarrow c$ finite ($\Rightarrow 2mc^2$ available)

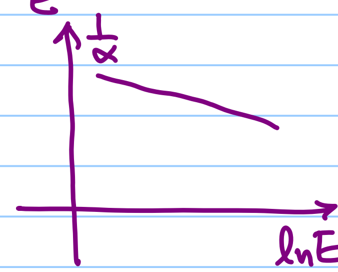
- Pair-popping (e⁻) (e⁺) for $\Delta t \sim \frac{\hbar}{\Delta E} \sim \frac{\hbar}{mc^2} \sim 10^{-21}$s for e^-, e^+

- Consider strength of $\vec{E}$ field of, say, e^-.

- 

  - Cloud of virtual e^+–e^- dipoles
  - Polarized by e^- field ($\vec{E}$)
  - Virtual dipoles screen $\vec{E}$

  $\Rightarrow \frac{\partial}{\partial E}\alpha_{EM} > 0$ i.e.



[Running of couplings of SM
$\Rightarrow$ unification 'optional'...]

- In string context, want conformal invariance in quantum theory: full Virasoro algebra of L_m's.

(3.1a) $\Rightarrow$ $\boxed{\beta_{g_{\mu\nu}} = 0}$

b $\quad \boxed{\beta_{B_{\mu\nu}} = 0}$

c $\quad \boxed{\beta_{\Phi} = 0}$

Techniques used:
- Background field method
- Riemann normal coörd expansion
- Loop diagrams in 2-d QFT (worldsheet perturbation thy)

$\boxed{\Rightarrow \text{equations of motion for spacetime fields.}}$

i.e. Not any $\{g, B, \Phi\}$ is consistent !!

Reconstruct action for spacetime fields.

Common Sector: $\forall$ superstrings is $\{g_{\mu\nu}, B_{[2]}, \Phi\}$
For IIA & IIB also see R-R fields $\{C_{[n]}\}$, n odd/even;
heterotic also have gauge fields $\{A_{[1]}{}^{I}\}$;

type I have $\{A_{[1]}\}$ ← non-abelian.

+ fermions

$\Phi \to 0$ at ∞. Φ_∞ is g_s

(4.1) $$\boxed{S_{NS}^{(bos)} = \frac{1}{16\pi G_{10}} \int d^{10}x \sqrt{-g}\, e^{-2\Phi}\left(R + 4(\nabla\Phi)^2 - \frac{1}{2}|dB_{[2]}|^2\right)}$$

where $dB_{[2]} \equiv H_{[3]}$ and $|d\omega_{p-1}|^2 = \frac{1}{p!}(d\omega_{p-1})^{\mu\nu\ldots\alpha}(d\omega_{p-1})_{\mu\nu\ldots\alpha}$

(4.2) where $\boxed{16\pi G_{10} = (2\pi)^7 g_s^2 \ell_s^8}$ is derived here ☺

- Can transform to Einstein metric $\tilde{g}_{\mu\nu}$ from what string couples to, hence called "string metric" $g_{\mu\nu}$

(4.3) $$\boxed{\tilde{g}_{\mu\nu} = e^{-4\Phi/(D-2)}\, g_{\mu\nu}}$$

Then
$$\boxed{S_{NS}^{(bos)} = \frac{1}{16\pi G_{10}} \int d^{10}x \sqrt{\tilde{g}} \left\{ \tilde{R} - \frac{1}{2}(\tilde{\nabla}\Phi)^2 - \frac{1}{2}|\tilde{d}B_{[2]}|e^{-\Phi} \right\}}$$

(*) ⇒ "right-sign" action for dilaton

'Gauge' fields
The other non-common sector depends on the superstring theory in question. We'll do IIA for definiteness – this is the nonchiral $\mathcal{N}=2$ SUSY theory.

(4.4)
(no D8's here)
$$\boxed{\begin{aligned} S_{R-R}^{(bos)}(\text{IIA}) = \frac{1}{16\pi G_{10}} \Big[\int d^{10}x \sqrt{-g} \Big\{ &-\frac{1}{2}|dC_{[1]}|^2 \\ &-\frac{1}{2}|dC_{[3]} - H_{[3]} \wedge C_{[1]}|^2 \Big\} \\ &+ \int B_{[2]} \wedge dC_{[3]} \wedge dC_{[3]} \Big] \end{aligned}}$$

(4.5) Open string: SYM for $A_{[1]}$ and $\boxed{g_{YM}^2 = (2\pi)^{p-2} g_s \ell_s^{p-3}}$

↗ See also my TASI notes

THE DARK SIDE OF STRING THEORY

hep-th/0008241

GR with action

$$S = \frac{+1}{16\pi G_N}\int d^Dx\sqrt{-g}\,R$$

has Schwarzschild <u>black hole</u> solutions, which solve vacuum Einstein eqns

(5.1) $$R_{\mu\nu} - \tfrac{1}{2}g_{\mu\nu}R = 0$$ } Nonlinear eqn: support nonzero M_{ADM} although $T_{\mu\nu}=0$

Couple to matter via rough-&-ready rule (for bosons)

$$\eta_{\mu\nu}\to g_{\mu\nu}$$
$$\partial_\lambda \to \nabla_\lambda$$

(Need $\partial\to\nabla = d + \omega_{ab}\Gamma^{ab}$ for fermions)

and

$$T_{\mu\nu} \equiv -\frac{2}{\sqrt{-g}}\frac{\delta\mathcal{L}_{matter}}{\delta g^{\mu\nu}}$$

includes measure-piece $\sqrt{-g}$, typically

e.g. scalar field $\mathcal{L}^s = -\tfrac{1}{2}\sqrt{-g}\,g^{\mu\nu}\nabla_\mu\phi\nabla_\nu\phi = -\tfrac{1}{2}\sqrt{-g}\,(\partial\phi)^2$

EM field $\mathcal{L}^{em} = -\tfrac{1}{4}\sqrt{-g}\,g^{\mu\nu}g^{\alpha\beta}F_{\mu\alpha}F_{\nu\beta}$

so $$T^s_{\mu\nu} = +\partial_\mu\phi\,\partial_\nu\phi - \tfrac{1}{2}g_{\mu\nu}(\partial\phi)^2$$

$$T^{em}_{\mu\nu} = -F_{\mu\lambda}F^\lambda{}_\nu + \tfrac{1}{4}g_{\mu\nu}F^{\lambda\sigma}F_{\lambda\sigma}$$

We can search for solutions to SUGRA action (4.4)!

- Simplest solutions are <u>Stable p-branes</u> (<u>half susy unbroken</u>)

(5.2) satisfy $\boxed{\text{tension} = c(g_s)\cdot|\text{charge}|}$ ← higher-d version for p-form potentials

<u>"BPS states"</u>

B charge

(5.3) e.g. F1 $\boxed{\tau_{F1} = \frac{1}{2\pi\ell_s^2}}$ $(\ell_s = \sqrt{\alpha'})$ NS5 $\boxed{\tau_{NS5} = \frac{1}{(2\pi)^5\ell_s^6 g_s^2}}$

(5.4) Dp $\boxed{\tau_p = \frac{1}{(2\pi)^p\ell_s^{p+1}g_s}}$

(5.5) → RR charge B charge

$Q_{NS5} = \int_{S^3} e^{-2\Phi} dB_{[2]}$

Conserved charges carried?

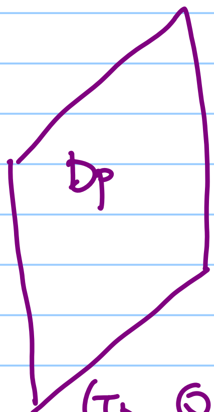
(6.1) $$Q_{F1} = \int_{S^7} *\left(e^{-2\Phi} dB_{[2]}\right)$$

$\int$ up flux through $\perp$ S^{8-p}

(6.2) $$Q_{Dp} = \int_{S^{8-p}} *\left(dC_{[p+1]}\right)$$

$\{t, \vec{x}_{\parallel}, r, \Omega_{8-p}\}$

$[1+p+1+(8-p)=10]$ ☺



(τ_{Dp}, Q_{Dp})
R-R charge
Couples naturally to $C_{[p+1]}$

F1
(τ_{F1}, Q_{F1})
NS charge
Couples naturally to $B_{[2]}$

particle
• (m, q)
EM charge
Couples naturally to $A_{[1]}$

By solving eqns of motion for $\{g_{\mu\nu}, B_{[2]}, \Phi, C_{[n]}\}$, find

(6.3) $$\boxed{\begin{aligned} &B_{[2]} = 0 \qquad\qquad \text{Dp} \\ &e^{\Phi} = H^{(3-p)/4} \\ &C_{[p+1]} = \left(\frac{1}{H} - 1\right) dx^0 \wedge dx^1 \wedge \dots \wedge dx^p \\ &ds^2 = \frac{1}{\sqrt{H}}\left(-dt^2 + d\vec{x}_{\parallel}^{\,2}\right) + \sqrt{H}\left(dr^2 + r^2 d\Omega_{8-p}^2\right) \end{aligned}}$$

where

(6.4) $$\boxed{H = 1 + c_p g_s N \left(\frac{\ell_s}{r}\right)^{7-p}}$$

(6.5) $$c_p = \frac{(2\sqrt{\pi})^{5-p}}{\Gamma((7-p)/2)}$$

Possesses SUSY ; $\Leftrightarrow$ Projection condition on spinors.

For F1,

(7.1)
$$\boxed{\begin{aligned} ds^2 &= H^{-1}(-dt^2+dx^2) + dr^2 + r^2 d\Omega_7^2 \\ B_{[2]} &= \left(\frac{1}{H}-1\right) dt\wedge dx \\ e^{\Phi} &= H^{-1} \end{aligned}} \quad \text{F1}$$

(7.2)
$$\boxed{H = 1 + c_1 N \left(\frac{l_s}{r}\right)^6 g_s^2}$$

while for NS5

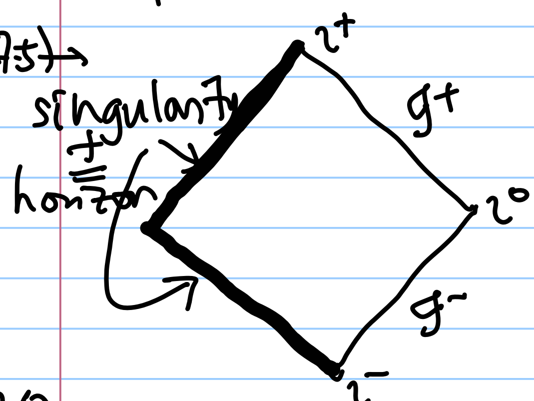
(7.3)
$$\boxed{\begin{aligned} ds^2 &= -dt^2 + d\vec{x}_{\parallel}^2 + H\left(dr^2 + r^2 d\Omega_3^2\right) \\ H_{[3]} &= Q\,\epsilon_{[3]} \\ e^{\Phi} &= H \end{aligned}} \quad \text{NS5}$$

(7.4)
$$\boxed{H = 1 + c_5 N \left(\frac{l_s}{r}\right)^2}$$

- Causal structure?
  - light-cones always at 45° on Penrose diagram
  - bring ∞ $\mathcal{M}$ to finite place via coörd transf.

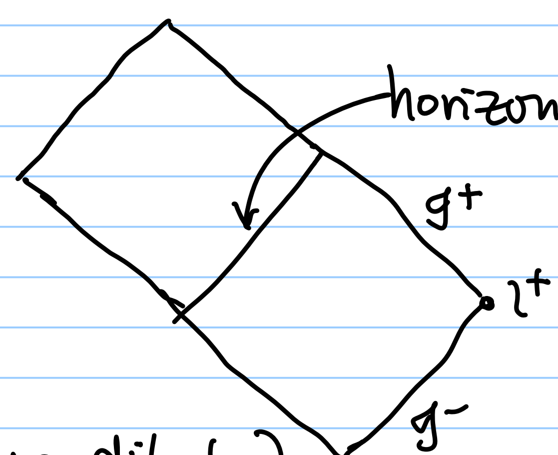
$Dp \neq 3$, F1

(7.5) →



(7.6) →

D3



(For NS5: geometry → $S^3 \times$ linear dilaton)

In GR, when horizon does not 'properly' cover singularity, questionable as to whether can call it censored - is the singularity really naked??

Compute geodesic eqn $p^\mu \nabla_\mu p^\nu = 0$; for photon

(8.1) $$\frac{dp^\mu}{d\lambda} + \Gamma^\mu{}_{\nu\lambda} p^\nu p^\lambda = 0$$ where λ = affine param.

Discover

(8.2) $\otimes$ for $p \lesssim 5$, photon takes $\Delta\lambda \to^{+} \infty$ to reach g^+ $\Rightarrow$ censored;
for $p = 6$, naked

[for $p = 7, 8, 9$: different story $\because$ not AF & $\therefore$ N limited $\Rightarrow$ SUGRA ~]

- Singularity can be resolved in a way I'll describe in the last lecture or two.

- Important : D3 geometry as $r \to 0 \longrightarrow AdS_5 \times S^5$
NS5 " " " $\begin{Bmatrix} \text{linear} \\ \text{dilaton} \\ \text{warp} \\ \text{factor} \end{Bmatrix} \times S^3$

(r, Ω) are isotropic coords. They also happen to be harmonic coords, i.e. $\Box x^\alpha = 0$, but this FAILS for p-brane spacetimes in general because introducing non-extremality makes isotropic coords (for which $g_{\Omega\Omega} = g_{rr} \cdot r^2$) non-harmonic.

! Important to note this fact for calculating M_{ADM} in more general spacetimes! in string theory SUGRA geometries.

- ▷ Hawking radiation
- ▷ Bekenstein-Hawking entropy
- ▷ Reissner-Nordström, $M = |Q|$ BPS & $AdS_2 \times S^2$
- ▷ Solution generating : algebraic technique
- ▷ W solution & relationship to NS-NS BH

$\Rightarrow$ as desired, switch to TASI notes ☺