Today:

Holography

and

Singularity Resolution

Singularities

Classical string/M theory = SUGRA.

No-hair theorems (even if they exist, which is not the general case, at least in $d = 5$ –) break down if singularities naked,
$\longrightarrow$ whole geometry in question.

Both clothed singularities (spacelike, null, timelike),
and naked ones, occur in *classical* string/M theory.

Tool for showing causal structure of spacetime: Penrose diagrams.

[Carter-]Penrose diagram gives causal structure of spacetime manifold.

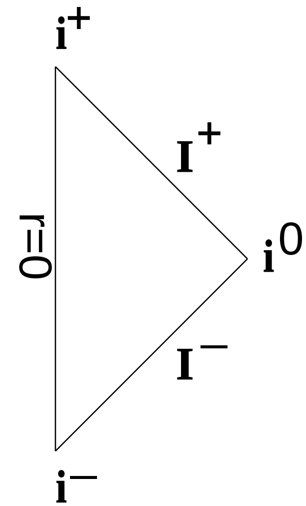
Use conformal transformation to bring infinity onto the page.

Light-beams go at 45°; timelike geodesics steeper, spacelike less so.

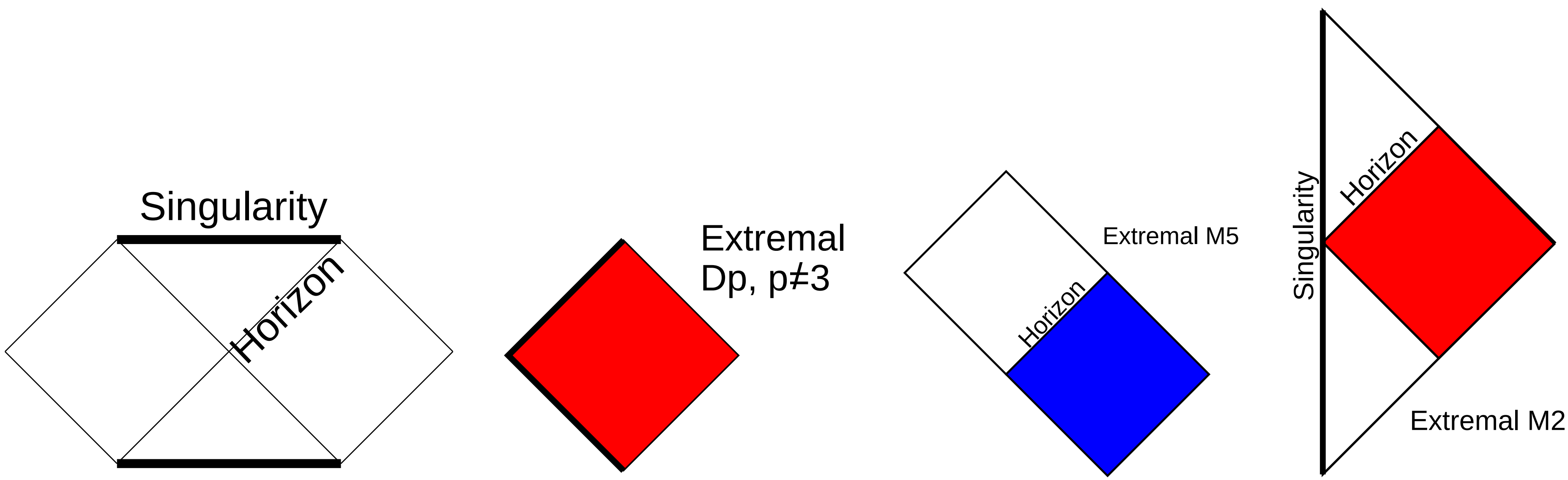
Penroses (nice-smelling??)

By tradition,
only (t, r)-plane drawn;
for $d = 3 + 1$ have transverse S^2.

Minkowski space: $\longrightarrow$



Various braney spacetimes appearing in string/M theory:

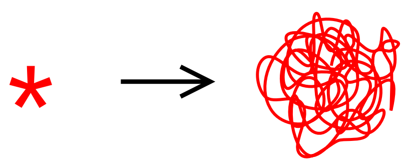


How to Fix Singularities

Horowitz+Myers '95: Quantum string theory can fix singularities by:

- Smoothing them out;
  spacetime loses meaning at $\geq$ stringy curvature,
  other degrees of freedom take over.

- Ruling them out *ab initio*.
  Some spacetimes are so sick that they must be considered
  figments of a classical physicist's imagination .
  e.g. $M < 0$ Schwarzschild is *unphysical* .



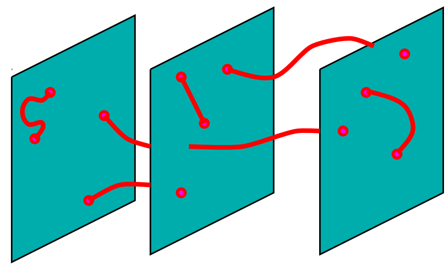
Reasoning: if a resolution of it did exist, the vacuum would become unstable!

For this reason: always be extremely careful if you rely on a naked spacetime singularity for a physics result. My advice is not to!

Building Gauge Theories and Spacetimes

D-brane $\equiv$ hypersurface on which open strings end (must, in type II).
String endpoints carry gauge labels for free; keep track of which brane.

e.g.
N=3



Gives rise to U(N) gauge theory stuck on the branes.
Gauge coupling is a *derived* quantity :

$$\boxed{g^2_{\mathsf{YM}} \sim g_{\mathsf{s}}\,\ell_{\mathsf{s}}^{p-3} \qquad (d=p+1)}\,.$$

Can also see this from:

Groundstate of open string $\supset$ spin-1 gauge field .

Groundstate of closed string $\supset$ spin-2 graviton .

Gravitational constant also *derived* :

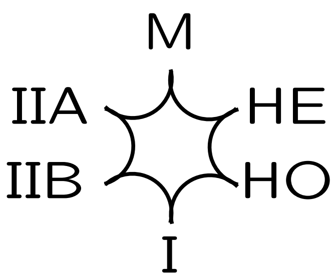
$$\boxed{G_{\mathsf{N}} \sim g_{\mathsf{s}}^2 \ell_{\mathsf{s}}^8 \qquad (d=10)}\,.$$

Tension of string 1 in ℓ_{s} units, D-brane $1/g_{\mathsf{s}}$.

Extreme String Theory

Dial-twiddling is a very profitable enterprise – still going strong!

Tons of “money” made by taking extreme limits - that was a theme in most of the duality revolution!



$\longleftrightarrow$ M=? $\longleftrightarrow$

Matrix Theory
AdS/CFT
NCSYM,
NCOS

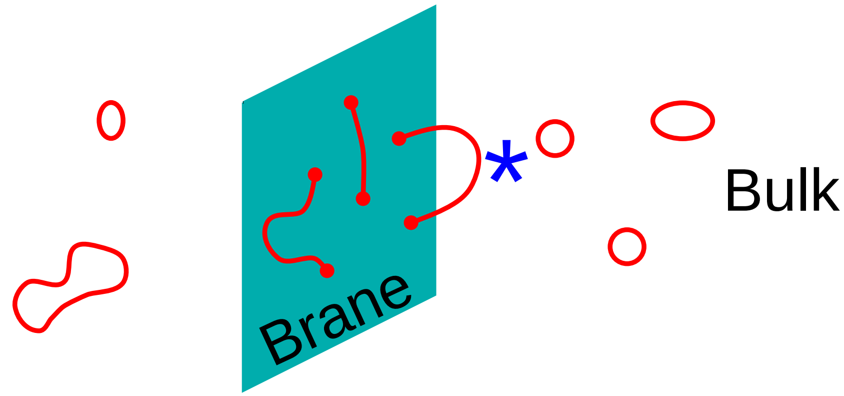
Appropriate to try taking things to extremes: we are

Building models of low-energy particle physics and cosmology

Understanding of full theory is as yet incomplete!

Decoupling Engineering

Dial limit: $\ell_s \to 0$ and keep g^2_{YM} fixed.



N Dp-branes in Type IIA/B string theory

↙ ↘

Brane: gauge theory

↓

Engineering $d = p + 1$ SYM on the branes.

⊛

Bulk: black p-brane

↓

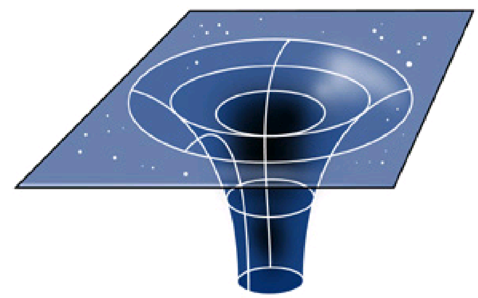
Going near-horizon in the $d = 10$ spacetime.

⊛: Coupling between bulk and brane vanishes. Makes correspondence possible. *No* gravity on the brane (contrast Randall-Sundrum!).

Black Branes and Dial-Twiddling

Black p-branes = analogs of black holes. ISO(1,p) symmetry along p brane dimensions. Transverse space has $10-(p+1)=(9-p)$ dimensions. $\rightarrow$ Gauss's Law gives potentials $1/r^n, n=(7-p)$.

In analogy to Schwarzschild, corrections to flat metric $\eta_{\mu\nu}$:



$$\delta g_{\mu\nu} = (G_{\rm N})\left[\frac{\text{Mass}}{\text{Vol}}\right]\frac{1}{r^n}$$
$$= (g_{\rm s}^2\ell_{\rm s}^8)\left[\frac{N}{g_{\rm s}\ell_{\rm s}^{p+1}}\right]\frac{1}{r^n} = \frac{g_{\rm s}N\ell_{\rm s}^n}{r^n}\,.$$

Limit #1: take $g_{\rm s}\rightarrow 0$ but fix $\ell_{\rm s}, r, N$.

Effect on metric: $\delta g_{\mu\nu}\rightarrow 0$. Flat spacetime.

Limit #2: take $\ell_{\rm s}\rightarrow 0$, but fix $g^2_{\rm YM}N$ and $U\equiv r/\ell_{\rm s}^2$.
Effect on metric:

$$\delta g_{\mu\nu} = \frac{1}{\ell_{\rm s}^4}\,\frac{g^2_{\rm YM}N}{U^n}\rightarrow\infty\,.$$

Asymptotically flat ($\eta_{\mu\nu}$) part of geometry lost.

Going near-horizon in the $d = 10, 11$ spacetime yields what?

Branes	Spacetime
D3	$\mathrm{AdS}_5 \times \mathrm{S}^5$
D1+D5	$\mathrm{AdS}_3 \times \mathrm{S}^3 (\times \mathrm{R}^4)$
M2	$\mathrm{AdS}_4 \times \mathrm{S}^7$
M5	$\mathrm{AdS}_7 \times \mathrm{S}^4$

Conformal group of $\mathrm{CFT}_{d=p+1}$	Isometry group of AdS_{p+2}

AdS weakly coupled when CFT strongly coupled,
and vice versa = anatomy of a strong/weak duality.
Classical $\longleftrightarrow$ Quantum.
Holographic!

Crossover between AdS and CFT depends on coupling parameters.
Need large-N to get a SUGRA region.

D3-branes

Maldacena: Holographic Duality

$$\left\{\ \ell_s^2\mathcal{R}\ ,\ g_s\ \right\}\ \leftrightarrow\ \left\{\ (g_{YM}^2 N)^{-1}\ ,\ N^{-1}\ \right\}.$$

Classical $\leftrightarrow$ Quantum duality. Holographic .

Helps that Vol(AdS)$_n$ $\propto$ Area(AdS)$_n$ at ∞, but goes *much* deeper.

Many tests passed.

Witten: (AdS-Schwarzschild Black Hole) $\times$S^5 $\leftrightarrow$ Finite temperature in gauge theory

In-principle resolution of:

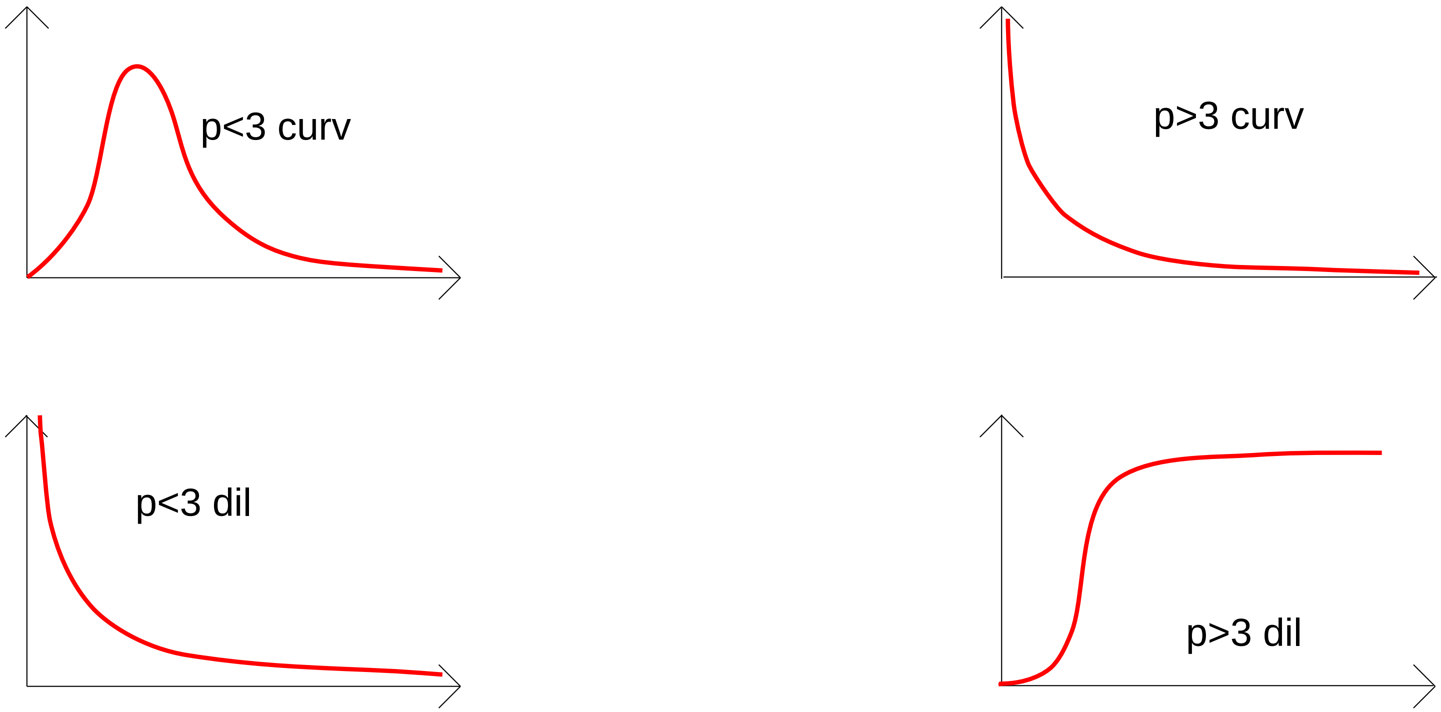
- black hole information problem;
- singularity.

Nature of strong/weak duality:

Only *one* description of the physics is perturbative in any given region of parameter space.

Where Dp-branes go bad

(For $p = 3$, that curvature and dilaton are both U-independent.)



Note interesting fact: if asymptotically flat part of geometry is removed, i.e. lose 1 in harmonic function H_p, then behaviour of both curvature and the derivative of dilaton becomes monotonic. This turns out to be a crucial SUGRA fact in context of Dp-brane gravity/gauge correspondences of [IMSY].

Dp-branes

To get $d = p + 1$ SYM, use Dp-branes.
Take low-energy limit: wiggly strings disallowed.

Gives U(N) SYM$_{p+1}$ on the branes. Gauge coupling $g^2_{\mathsf{YM}} \sim g_s \ell_s^{d-3}$.

Want to include physics of straight (non-wiggly) strings stretched between branes. Mass is

$$U \equiv \frac{r}{\ell_s^2}$$

Want to retain nontrivial brane physics in decoupling limit.
Keep SYM coupling and mass of stretched strings.
For dimensionless statement,

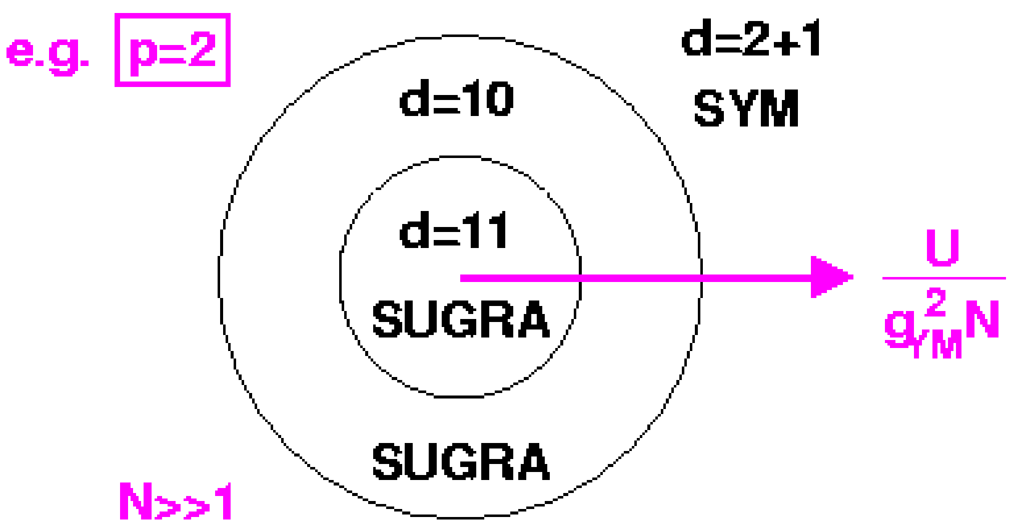
$$\text{fix} \quad g^2_{\mathsf{YM}} E^{p-3} \sim g_s (E\ell_s)^{p-3}$$

Case $p < 3$ gives relevant physics: no need for UV completion.
Case $p > 3$ gives irrelevant physics - so need extra degrees of freedom in UV - string theory tells us what these are.

In $d = 10$ spacetime, decoupling amounts to losing flat asymptopia.

For general Dp, curvature of spacetime varies with U.
Dilaton field e^{Φ} gives local value of string coupling.
Curvature and dilaton get strong in different places.
Crossovers depend on dimensionless combinations of parameters and U.
Gives *phase diagram picture* of correspondences.

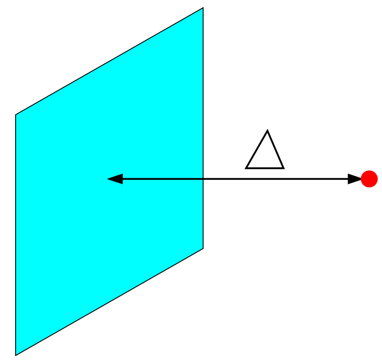
Investigate where different descriptions of the physics,
e.g. $\text{SUGRA}_{10,11}$ or SYM_{p+1}, are perturbative.
Typically get a CFT in only one part of phase diagram.



For $p = 2$: happens in middle for small-N: SO(8)-invariant $d = 2 + 1$ CFT.

No-hair / CMW Theorem duality

D$p+4$-branes (“big”) and
Dp-branes (“wee”)
are in neutral equilibrium
for any separation Δ.



Finding localized SUGRA solutions in hard in general!
- typically easier to have dependence on fewer coordinates
especially in systems with more than one type of brane.

Study SUGRA solutions:
can big-branes support wee-brane hair as $\Delta \to 0$?

Discover chracter of solutions depends strongly on p
- via strength of big-brane potentials which feed into wee-brane e.o.m.’s.

$-1 \leq p \leq 1$	$p \geq 2$
baldness	hair

In decoupling limit, $(E\ell_s) \to 0$.
This means big-brane SYM irrelevant c.f. wee-brane SYM.
Hence, working with $d = p + 1$ gauge theory.

Wee-branes can be thought of as instantons in big-branes.
Wee-dim gauge theory is theory of collective modes, including scale size of instanton, ρ.

Recall: classical SUGRA $\longleftrightarrow$ quantum SYM.

CMW theorem: in low-dimensional QFT, quantum fluctuations "feel out" whole space. IR problem, in $d = (-1, 0, +1) + 1$.

These IR quantum fluctuations make instanton size field

$$\langle \rho^2 \rangle \to \infty\,, \quad \text{for } p = -1, 0, +1.$$

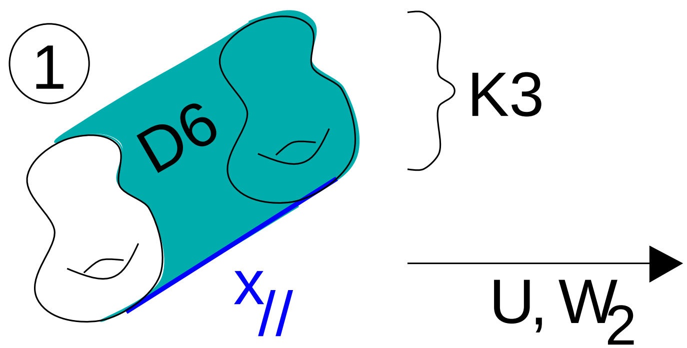
i.e. QFT delocalization matches SUGRA result.
QFT delocalization also matches *rate* as function of Δ! - e.g. power in $d = 0 + 1$, log in $d = 1 + 1$.

Moral: *baldness strikes only the smallest p-branes*

– or sometimes, SUGRA solutions don't exist for good QFT reasons.

Pure $\mathcal{N}=2$: the story of the Enhançon

Need to break half of SUSY. Many dual realizations.
Easiest (we found) was to wrap N D6-branes on a K3 surface



Set Vol(K3)
out at ∞
to be $\equiv (2\pi R)^4$

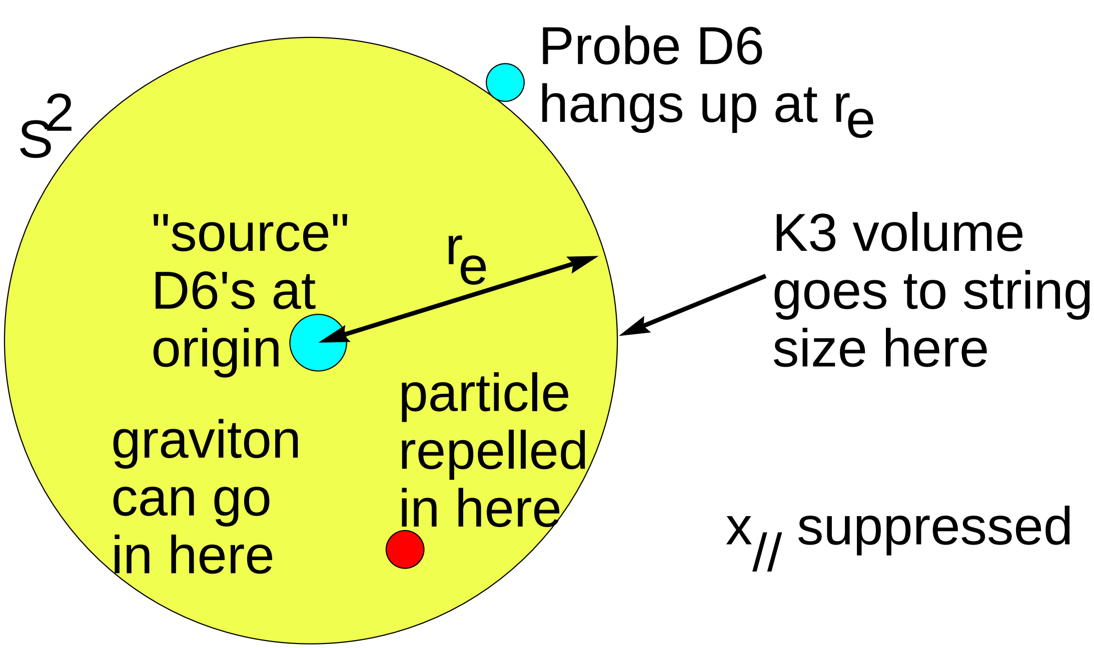
Wrapping N D6 on K3 surface gives what quantum #'s?

- D6-brane charge N
- Peculiarity of K3 curvature $\Rightarrow$ D2-brane charge $(-N)$, distributed evenly over the whole K3.

Guess SUGRA solution based on old experience.
Problem: obtain singularity.
Worse problem: singularity is naked - no horizon!! Dubbed *repulson*. .
Same type of singularity as interior of familiar Reissner-Nordstrøm black hole of $d=4$. But unsafe because singularity not clothed at all.

Probe technique: take a test D6-brane (wrapped on K3) and use it to "test out" (probe) the spacetime.

K3 volume *runs* in the spacetime: even though we set it to a constant at ∞, the D6's are heavy and their gravity fields warp the K3 volume. Discover that our friend the *stringy minimum distance phenomenon* prevents the probe D6 from getting closer than r_e, the *enhançon radius*! If it were to go any closer, the K3 volume would shrink even further and take the probe into the illegal negative tension regime.

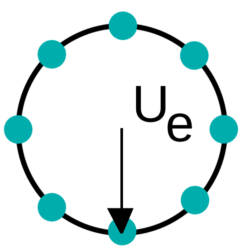


Exactly at $R = \ell_s$, get enhanced gauge symmetry SU(2). String duality says D6 wrapped on K3 dual to "W-boson" string wrapped on circle. Hence "enhançon".

So what is the real story of the enhançon's SUGRA geometry?
First - the original guess was just that - the geometry was nakedly singular so there is *no* guarantee it was the correct one carrying the required quantum numbers.
Clue to salvation lies in dependence of the enhançon radius r_e on N:

$$r_e \propto N$$

Consequence?
Well, let's see if we *can* put all the D6's in the middle of spacetime to make that nasty naked singularity. Let's do it step-by-step:
1: Put 1st D6 there. $r_e(1)$.
2: Try to bring up 2nd D6. Stops at $r_e(1)$; can't get closer.
3: Try to bring up 3rd D6. Stops at $r_e(2) = 2r_e(1)$.
4...N: Enhançon radius keeps growing and end up with a *Dyson sphere* of source D6-branes!



(Lovely dual picture in strongly coupled gauge theory - Seiberg-Witten curve – breaking U(N)→U(1)^N.

Simple fact: naked singularity never appears because string theory doesn't allow it to be built.

Interesting additional piece of physics – SUGRA is smart enough to know about the enhançon radius! How so? It's just a low-energy approximation to the full string theory – and in particular has no knowledge of string winding states or enhanced gauge symmetry!

Well, if have piece of curved space joined to piece of flat space, must have source of energy-momentum (and charge etc. also) to ensure matching conditions are satisfied.

1. There is so much SUSY in this problem that the source D6-branes can actually sit at any radius $r \geq r_e$.

2. If we tried to put the Dyson sphere radius at $r < r_e$, then SUGRA would require us to use negative-tension branes.

3. Negative-tension objects which can fluctuate* are a *disaster*. Reason: any fluctuation *de*creases the energy. Even the vacuum becomes unstable under such conditions (pair production).

*Orientifolds are OK - can't fluctuate by symmetry

Singularities and dimension

Important note: whether a spacetime geometry is singular depends on dimension of SUGRA theory it is embedded in. Some spacetimes singular in lower-d are nonsingular when lifted to higher-d.

E.g. D6-brane: in $d = 10$, null horizon. Actually, photon emanating from singularity makes it out to infinity in finite affine parameter! But lifted to $d = 11$, nonsingular gravitational object.

For understanding possible resolution of singularities in terms of basic stringy objects like D-branes, best dimension to do singularity analysis is d=10, which is dimension in which D-branes naturally live. Generally more confusing to try to do analysis directly in lower dimensions. Also, need to be sure that any operation you do in SUGRA also makes sense in string theory...