[20] QUESTION 1: MULTIPLE CHOICE

In each part, select the response(s) which *best* complete(s) the statement or answers the question. Partial marks will be given, where possible, for partially correct answers. Wrong answers will get a **-1** mark penalty, so if you haven't got a clue it's best to leave a question unanswered than to guess.

[5] 1A: In thermal physics, states *accessible* to a system $\mathcal{S}$ are:
(i) those states in which $\mathcal{S}$ could find itself, on the timescale of the experiment ;
(ii) those states of $\mathcal{S}$ which have energy $\mathcal{E}$ no higher than the thermal energy $k_B T$;
(iii) those states with the right multiplicities ;
(iv) those states satisfying the constraints on $\mathcal{S}$;
(v) Exactly two of the above must be true (state which two).

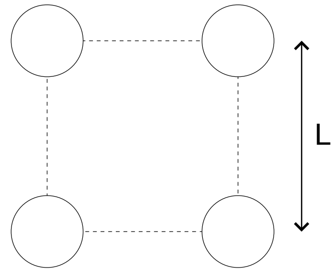
[5] 1B: Which of the following will tend to *decrease* the entropy?:
(i) removing a constraint internal to a system;
(ii) straightening a folded protein;
(iii) molecular dissociation;
(iv) a reversible compression of an ideal gas;
(v) only two of the above (state which two).

[5] 1C: A powerful laser is set up such that a pulse of 50mJ is trapped between two mirrors 0.75m apart, and the beam has a 4mm diameter. If a blackbody were to produce the same total energy density (U/V) as this system, what would the blackbody temperature be?:
(i) of order 10K or less ;
(ii) of order a hundred Kelvin ;
(iii) in the thousands Kelvin ;
(iv) in the tens of thousands Kelvin ;
(v) above a hundred thousand Kelvin .

[5] 1D: Make the (rather unrealistic) assumption that the air of the Earth's atmosphere is an isothermal ideal gas in a constant gravitational field. If in Banff, altitude 1475m, there is only 84% as much atmospheric pressure as at sea level, how little atmospheric pressure is there atop Sagarmatha (Mount Everest), altitude 8848m?:
(i) about 21% of sea level
(ii) about 27 % of sea level
(iii) about 35% of sea level
(iv) about 41% of sea level
(v) there is not enough information to solve the problem.

[20] QUESTION 2: ION CRYSTAL

Imagine a four-site square crystal, lying in a plane.



Suppose you are given two sodium ions (Na^+) and two Chlorine ions (Cl^-), to put into the four sites any way you like.

[8] (a) Enumerate the possible microstates of the system. How many microstates are there? Which of these microstates have the same energies?

Now, what are the two *macro*states of the system, and their degeneracies? Write an expression for the energy difference Δ between these two macrostates, in terms of α/L.

Hint: You will need the fact from electrostatics that the electric potential energy between any two charges separated by distance r is

$$U_{\text{electric}} = \frac{\alpha g_1 g_2}{r}$$

where g_1, g_2 are the charges in units of the electronic charge, and α is some constant.

[6] (b) Move the zero of potential energy, so that the lower-energy level has energy zero, and so the energy of the higher-energy level is Δ. Compute the partition function Z as a function of absolute temperature τ, and Δ. Show that the average energy is

$$U = \frac{2\Delta e^{-\Delta/\tau}}{1 + 2e^{-\Delta/\tau}}$$

Show all steps in your working.

[6] (c) Compute the heat capacity $C_V(\tau, \Delta)$.
Or, if algebra gives you a stomachache, draw a graph of U as a function of τ and from that draw a graph of C_V (label your axes!).

[Bonus] Briefly, contrast the thermodynamic behaviour of this system with a system of phonons. Comment on the limits of your formulæ at high and low temperatures, and on dimensional analysis.

[20] QUESTION 3: IDEAL GAS OF "PEETONS"

Consider a make-believe gas, consisting of relativistic "peeton" wavicles, confined to a two-dimensional[1] square box of area $A = L^2$. Assume for the purposes of this problem that the number of peetons N is *fixed*, and that the speed of light is $c = 2,000\text{ms}^{-1}$. Assume that the relativistic energy levels for a peeton are given by

$$\mathcal{E}_{\vec{n}} = \frac{\hbar c \pi}{L} |\vec{n}|$$

The vector $\vec{n} = (n_x, n_y)$ describes how many half- de Broglie wavelengths of the wavicle fit into the box in directions (x, y), and $n_x, n_y \in \{1, 2, 3 \ldots\}$

[6] (a) For parts (b) and (c), we will need to know that it is OK to treat the spacing between adjacent energy levels as essentially continuous. Find the energy spacing $\Delta\mathcal{E}$ between the two lowest energy levels for a peeton, in terms of $L, c, \hbar$. It will be OK to replace sums by integrals in computing the single-peeton partition function Z_1, if we choose this spacing to satisfy

$$\Delta\mathcal{E}/\tau < 10^{-6}$$

where τ is the fundamental temperature. If our square box has 10cm sides, how big must the temperature T be to satisfy this condition?

[6] (b) The partition function for a relativistic peeton in *one* dimension (1-d) is different than the nonrelativistic case you are familiar with. Assume that

$$Z_1(\text{1-d}) = \frac{\tau}{\hbar c \pi} L\,.$$

Deduce the partition function for a single peeton in our *two*-dimensional (2-d) box, in terms of $A, \tau, \hbar, c$. Explain each part of your reasoning.
Now deduce the total partition function Z_N for N peetons in 2-d, in terms of $A, \tau, \hbar, c$. Explain each part of your reasoning. You should find that

$$Z_N(\text{2-d}) = \frac{1}{N!}\left[\left(\frac{\tau}{\hbar c \pi}\right)^2 A\right]^N$$

[8] (c) From Z_N, find the Helmholtz free energy F, for our gas of N peetons in the square box. Also, find the energy U, and the pressure p. Deduce the peeton ideal gas relation, which links p, A and τ.

[Bonus] Find the entropy σ as well, and contrast this plus all of your results in part (c) with what you know about regular ideal gases.

[1] We could achieve two dimensions experimentally by having an essentially monatomic layer of gas acting as lubricant between two blocks of solid.