What about the classical limit?

We can do this by taking

$$f_c = \frac{1}{e^{(\varepsilon-\mu)/\tau} \cancel{\pm 1}} \rightarrow e^{-(\varepsilon-\mu)/\tau}$$

Then $\tau \frac{\partial f_c}{\partial \mu} = e^{-(\varepsilon-\mu)/\tau} = \langle N \rangle_c$

so $\frac{\langle (\Delta N)^2 \rangle_c}{\langle N \rangle_c^2} = \frac{1}{\langle N \rangle_c}$ This is the ideal gas result.

$\boxed{\text{Superfluidity}}$

Let's pretend that the condensed phase (e.g. ^4He) is describable as a Bose-Einstein condensate.

The N_0 atoms have $\varepsilon = 0$

and hence $\vec{p} = \vec{0}$

and it's almost like there's nothing there...

In terms of viscosity, these N_0 atoms don't contribute at all

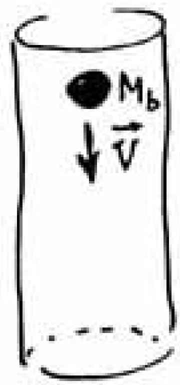
– unless you kick one (or some) up into higher (excited) orbitals

So we're saying that friction, which you think of as a macroscopic thing, has microscopic origins.
In this case, it would correspond to (e.g.)
^{4}He ($\varepsilon = 0$) hits wall, gets deflected, changes momentum, gets excited ...

In fact, the superfluid is not a bunch of free particles. As with many other many-body systems, the superfluid has a bunch of collective modes - excitations which belong to the whole system.
In crystals these excitations are sound waves
In superfluids they are longitudinal sound waves.
These waves, quantized, are called <u>quasiparticles</u>
Quasiparticles are not like electrons - they are not present if you break up the many-body system. But for low-energy physics of many-body systems working with quasiparticles is extremely productive as a framework for thinking

So how can you see this stuff in superfluids?

Get ^4He extremely cold & gone superfluid



Get a very smooth cylinder & fill it with superfluid. Drop a smooth ball.

In order to make an excitation (quasiparticle) need to conserve energy & momentum

(1) $$\frac{1}{2} M_b \vec{v}^2 = \frac{1}{2} M_b (\vec{v}')^2 + \underbrace{\varepsilon_{\vec{k}}}$$

quasiparticle energy

(2) $$M_b \vec{v} = M_b \vec{v}' + \underbrace{\hbar \vec{k}}$$

quasiparticle momentum

Are these equations always satisfiable?

Let's get rid of $\vec{v}'$:

(2) $$\vec{v}' = \vec{v} - \frac{\hbar \vec{k}}{M_b}$$ Substitute into (1) -

$$\Rightarrow (1) \quad \frac{1}{2} M_b \vec{v}^2 = \frac{1}{2} M_b \left(\vec{v} - \frac{\hbar \vec{k}}{M_b} \right)^2 + \varepsilon_{\vec{k}}$$

i.e.

$$\cancel{\tfrac{1}{2}M_b\vec{v}^2} = \cancel{\tfrac{1}{2}M_b\vec{v}^2} + \frac{\hbar^2\vec{k}^2}{2M_b} - \hbar\vec{v}\cdot\vec{k} + \varepsilon_{\vec{k}}$$

so $$\hbar\vec{v}\cdot\vec{k} = \varepsilon_{\vec{k}} + \frac{\hbar^2\vec{k}^2}{2M_b}$$

Simplification: take M_b to be large compared to $\hbar^2\vec{k}^2$. then

$$\hbar\vec{v}\cdot\vec{k} \cong \varepsilon_{\vec{k}}$$

The meaning of this?

- if $\vec{v} = \vec{0}$, nope! No "friction"

when $\vec{v}$ reaches right amount, will satisfy eqn.

need $$\boxed{\vec{v} \parallel \vec{k} \quad \text{and} \quad v_c = \frac{\varepsilon_{\vec{k}}}{|\hbar\vec{k}|}}$$

So what happened?

faster ball $\longrightarrow$ slower ball + quasiparticle

Can think of this as a scattering process (!)

We had for phonons

$$\varepsilon_{\vec{k}} = \hbar \vec{k} v_s \qquad \left(\varepsilon_{\vec{k}} = \hbar\omega_{\vec{k}} \,;\ \omega_{\vec{k}} = v_s \vec{k} \right.$$

v_s: speed of sound

rather than $c\vec{k}$ for photons)

So if this were the dispersion relation for quasiparticles in superfluid ^{4}He, we'd have

$$v_c = \frac{\varepsilon_{\vec{k}}}{|\hbar \vec{k}|} = v_s .$$

This is not actually what happens in ^{4}He; it's close conceptually though.

People really measure this stuff experimentally!

Last note: Unless $\tau = 0$,
below τ_E there is a normal fluid component too, and this does have viscosity.

So one has to take into account both effects.

[$v_c \approx 50\,\mathrm{m\,s^{-1}}$ from certain experiments
... check our large-M_0 approx.]