Brief review to refresh memory -

Fermions

$$f_{FD} = \frac{1}{e^{(\varepsilon-\mu)/\tau} + 1}$$

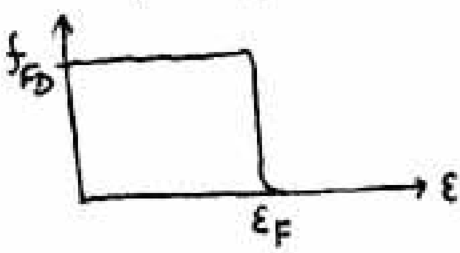
$$\mathcal{D}(\varepsilon) = \frac{V}{2\pi^2}\left(\frac{2m}{\hbar^2}\right)^{3/2} \varepsilon^{1/2}$$

(for spin-$\frac{1}{2}$)

$$\tau_F = \frac{\hbar^2}{2m}\left(3\pi^2 n\right)^{2/3}$$

At very low temperature

$\mu \cong \varepsilon_F$ and



"Fermion elbows"

$\Leftrightarrow$ large kinetic energy even at absolute zero

Degenerate Fermi Gas

Bosons

$$f_{BE} = \frac{1}{e^{(\varepsilon-\mu)/\tau} - 1}$$

$$\mathcal{D}(\varepsilon) = \frac{V}{4\pi^2}(2s+1)\left(\frac{2m}{\hbar^2}\right)^{3/2} \varepsilon^{1/2}$$

(for spin-s)

$$\tau_E = \frac{2\pi\hbar^2}{M}\left(\frac{1}{\zeta(\frac{3}{2})}n\right)^{2/3}$$

At very low temperature

$N(\varepsilon=0) \equiv N_0 \gg 1$

$$N_0 \cong N\left[1 - \left(\frac{\tau}{\tau_E}\right)^{3/2}\right]$$

$$N_e \cong N\left(\frac{\tau}{\tau_E}\right)^{3/2}$$

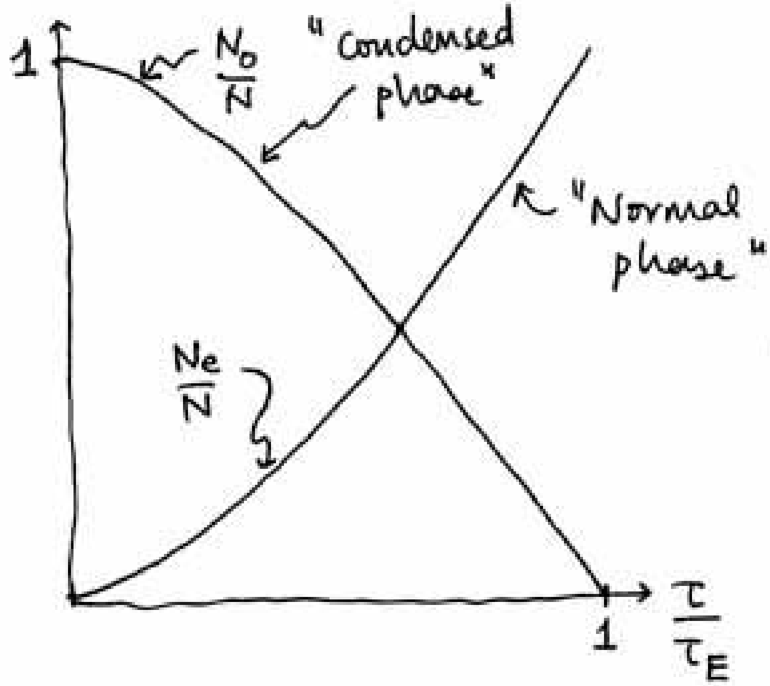
Almost all gas particles in ground orbital $(\varepsilon = 0)$

Bose-Einstein Condensation

Again using the $N_0 \gg 1$ approx,

find

$$N_0 \equiv N - N_e$$
$$\cong N\left[1 - \left(\frac{\tau}{\tau_E}\right)^{3/2}\right].$$



Experimentalists (& theorists) will refer to condensed & normal phases as if they're two different substances (They do behave very differently!)

$\boxed{\text{One Example: } {}^4He}$

How come ${}^4He$ is a boson?

Nucleus has 2 protons ($\Leftrightarrow$ is Helium)
2 neutrons (4 − 2)

both protons & neutrons are (composite) spin-$\frac{1}{2}$ fermions.

Rules of angular momentum addition
say

$s=\frac{1}{2}$ "+" $s=\frac{1}{2}$ $\rightarrow$ either $s=0$
or $s=1$ } bosonic

or <u>two fermions give a boson</u>

So with $2p^+$, $2n^0$ nucleons, ^4He <u>is</u> a boson.

What about ^3He? This has
$2p^+$, $1n^0$ $\Rightarrow$ fermionic nucleus.

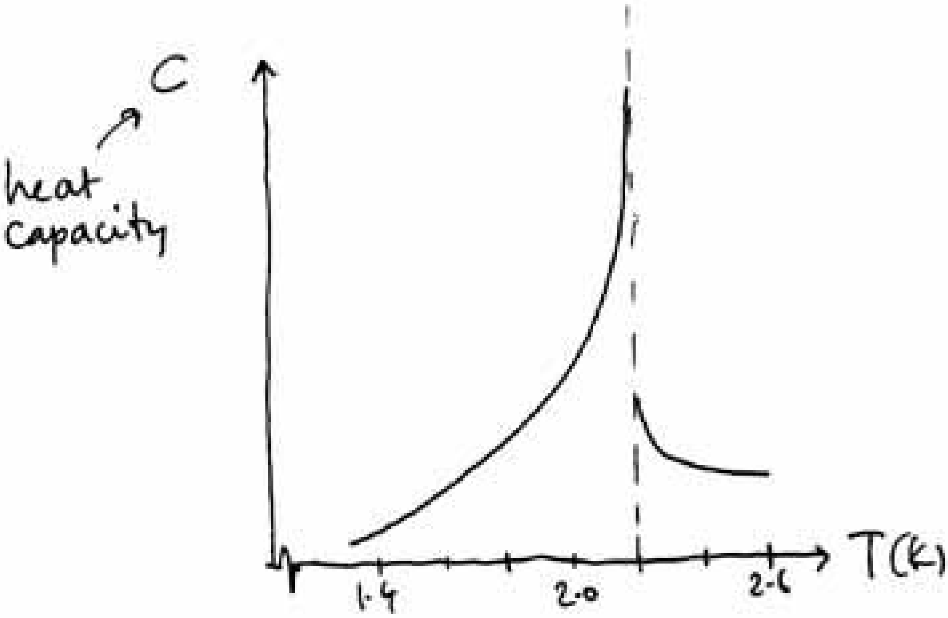
The ^3He's do a fun trick in order to get into the ground orbital - they <u>pair up</u>!!

Get ($^3He - ^3He$) condensing into ground state

Actually, electrons do this trick too, at low enough τ, becoming a (e-e) "Cooper Pair" in a metal - which is a mechanism for going superconducting.

(*) In truth, the real physics of superconductors and superfluids is MUCH more complicated than our simple BEC story here ... the only thing our theory really gets right is the condensation.

For liquid ^4He,



The shape of this graph looks like a λ, so it's called the "lambda point" transition

Below T_λ, the "condensed" phase is a superfluid, which has extremely low viscosity.

A bunch of superfluid can even "climb" the walls of a (very cold) beaker! (- by capillarity...)

It is also an extremely good thermal conductor

Superconductivity & related phenomena are a HUGE area of research, both theoretical and experimental.

About superfluid ^3He

Recall that

$$T_E = \left(\frac{2\pi\hbar^2}{M}\right)\left(\frac{N}{\zeta(\frac{3}{2})V}\right)^{2/3}$$

Now, $(^3He-^3He)$ is essentially a diatomic molecule. It is naturally going to take up more space than a single ^4He boson. Quite a bit more...

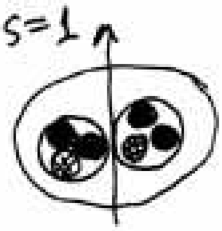
$$\Rightarrow T_E(^3He-^3He) < T_E(^4He)$$

(Mass factor $\uparrow$ also helps this).

Experimentally, transition temperature ~ few mK.

* Interestingly, for the 2 spin-$\frac{1}{2}$ ^3He fermions, when they pair up like this, they are in the $S=1$ angular momentum state!

$$\left(s=\tfrac{1}{2} \text{ "+" } s=\tfrac{1}{2} \rightarrow s=0 \text{ or } s=1\right)$$



Therefore, $m_s = -1, 0, +1$

and this system has magnetic properties

$\Rightarrow$ FUN :)

[DOUG OSHEROFF STORY]

As Geoff worked out in Monday's tutorial:

Fermions

$$\langle (\Delta N)^2 \rangle = \langle N \rangle (1 - \langle N \rangle)$$

$$\Rightarrow \frac{\langle (\Delta N)^2 \rangle}{\langle N \rangle^2} = \frac{1}{\langle N \rangle} - 1$$

$$= \frac{(1 - \langle N \rangle)}{\langle N \rangle}$$

$$\geq 0$$

because $0 \leq \langle N \rangle \leq 1$

For orbitals far below ε_F at low τ, fractional fluctuations are very small ($\to 0$) because $\langle N \rangle \to 1$!

Bosons

$$\langle (\Delta N)^2 \rangle = \langle N \rangle (1 + \langle N \rangle)$$

$$\Rightarrow \frac{\langle (\Delta N)^2 \rangle}{\langle N \rangle^2} = \frac{1}{\langle N \rangle} + 1$$

$$\geq 1$$

because $0 \leq \langle N \rangle < \infty$

At very low τ, BEC happens, and for ground orbital $\langle N \rangle \to$ large!

But for $\langle N \rangle \to$ large,

$\frac{\langle (\Delta N)^2 \rangle}{\langle N \rangle^2} \sim 1$ so

huge fluctuations (absolute) !

This is one more manifestation of the huge effect of spin & statistics of particles – at really low τ it matters a lot whether you have elbows or not.

What about the classical limit?

We can do this by taking

$$f_c = \frac{1}{e^{(\varepsilon-\mu)/\tau} \cancel{\mp 1}} \rightarrow e^{-(\varepsilon-\mu)/\tau}$$

Then $\tau \frac{\partial f_c}{\partial \mu} = e^{-(\varepsilon-\mu)/\tau} = \langle N \rangle_c$

so $$\frac{\langle(\Delta N)^2\rangle_c}{\langle N \rangle_c^2} = \frac{1}{\langle N \rangle_c}$$ This is the ideal gas result.

$\boxed{\text{Superfluidity}}$

Let's pretend that the condensed phase (e.g. ^4He) is describable as a Bose-Einstein condensate.

The N_0 atoms have $\varepsilon = 0$

and hence $\vec{p} = \vec{0}$

and it's almost like there's nothing there...

In terms of viscosity, these N_0 atoms don't contribute at all

– unless you kick one (or some) up into higher (excited) orbitals