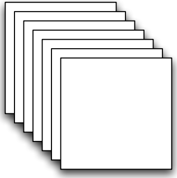
Origin of AdS/CFT

- AdS/CFT holography is an equivalence between gravitational (string) theories and *non*-gravitational field theories in one lower dimension. It gave the first concrete working models of the concept of holography for quantum gravity and grew out of studying nonperturbative D-branes in string theory.
- Consider a stack of N_p Dp-branes. Our key idea will be to have large N_p.



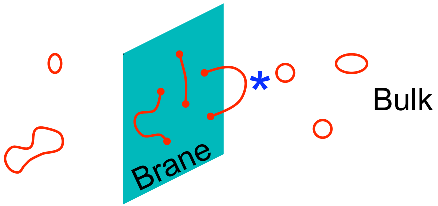
To see why, consider how the gravitational warping of a single Dp-brane scales. Corrections to the flat Minkowski metric scale as

$$\delta g_{\mu\nu} \sim \frac{(G_N)(\tau_{Dp})}{r^{7-p}} \sim g_s^2 \ell_s^8 \frac{N_p}{g_s \ell_s^{p+1}} \frac{1}{r^{7-p}} \sim \frac{g_s N_p}{(r/\ell_s)^{7-p}}$$

This shows that the variable controlling gravitational warping is $g_s N_p$.
- So we can obtain a seriously large warping of flat spacetime by having N_p become parametrically large while keeping g_s small.

Maldacena's decoupling limit

- Spacetime warping can be thought of as physics of closed strings. After all, the lowest state of excitation of a closed superstring includes the graviton mode. We can also look at the open string picture of Dp-brane physics.
- Lowest mode of open superstring has spin-one and mass zero. Theory living on worldvolume of N Dp-branes is $\mathrm{SYM}_{p+1} + \alpha'$ corrections $+\ g_s$ corrections. The variable controlling open string corrections when D-branes are present is actually $g_s N_p$, which is exactly what we found also controls spacetime warping.
- In general, dynamics of the full closed + open string sectors is unknown (c.f. string field theory). Maldacena's key insight was to take a very special 'decoupling limit' of this stack of N_p Dp-branes, which turns off interactions between closed strings in the bulk and open strings whose endpoints lie on the D-branes.



AdS=CFT

- In the decoupling limit, on the open string side you get $d = p + 1$ $U(N)$ gauge theory living on the Dp-brane worldvolume. On the closed string side, the decoupling limit zooms in on the *near-horizon* region of the geometry.
- Maldacena boldly realized that this decoupling limit meant that you could think about the stack of N_3 D3-branes *either* in terms of the near-horizon geometry *or* the worldvolume Yang-Mills theory. But because it is the *same* stack of D3-branes either way, those representations must be equivalent!
- The near-core geometries of D3, M2, M5, and D1+D5 systems produce $AdS_{p+2} \times S^{D-p-2}$ spacetimes, where D is the spacetime dimension, 10 in superstring theory and 11 in M theory. In brief: the S^5 factor arises for D3 because the r^2 in the transverse piece of the spacetime metric proportional to $d\Omega_5^2$ is cancelled off by the $H_3^{+1/2}$ factor, because for D3-branes the harmonic function scales as $1/r^4$ in the near-core region. To see the AdS_5 factor is only slightly more of a challenge. (You end up with the Poincaré patch of AdS_5.)
- Yang-Mills theory on flat D3-branes has $\mathcal{N} = 4$ SUSY. The vector field in the adjoint representation partners up with adjoint fermions and scalar fields. The field content of this theory is so special that it ends up having zero β-function. It is actually a *conformal* field theory. Hence, we say AdS=CFT.

Probes and IR/UV relations

- AdS/CFT gives a definition of quantum gravity in asymptotically AdS space in *Lorentzian* signature, via the duality and Wick rotation in the CFT.
- AdS/CFT sheds light on the question of background independence in quantum gravity. It is independent of bulk background except for its asymptotics which are locked down by boundary physics.
- One of the key ideas string theorists use every day is the concept of *probes*. Probes are objects that exist in the theory and are used to interrogate the physics. For fundamental string probes stretched between Dp-branes, field theory energy E is proportional to distance r between D-branes in bulk, with the constant of proportionality given by the string tension,

$$E = \frac{r}{2\pi\alpha'} \,.$$

  This is known as an *IR/UV relation* and is central to AdS/CFT. It says that the UV in the field theory is welded to the IR in the bulk and vice versa. It also makes clear that the *whole* bulk is what is dual to the whole CFT.
- Polchinski and I showed that gravitons have different IR/UV relations, which are easily understandable via solving a simple scattering equation in the bulk. Still, how deep it penetrates into the bulk is related to brane energy.

IMSY and applications of AdS/CFT

- For more general Dp-branes, you find that the harmonic functions conspire to *not* give you $AdS_{p+2} \times S^{D-p-2}$ spacetimes. Instead [for $p < 5$], you get a warped version where there is an r- (and p-) dependent conformal factor. What does this mean for the possibility of a correspondence like AdS/CFT? The 10D bulk spacetime background is only reliable for certain ranges of r, because either $\alpha' \mathcal{R}$ or e^{Φ} is monotonically growing as $r \to 0$ or $r \to \infty$. By dimensional analysis, the field theory coupling also runs with energy. 10D SUGRA geometry becomes unreliable when $g^2_{\rm eff}(E) := g_s N (\ell_s E)^{p-3}$ becomes of order unity, which is where the SYM field theory description takes over.
- This gives rise to the idea of a *phase diagram* for a D-brane/string system, which turns out to be a very generic and physically rich feature in holographic setups. There can be more than one description of the physics, but only one of them at a time can be weakly coupled.
- AdS/CFT has been applied to modelling quark-gluon plasma and cond-mat, with limited successes. Can calculate transport properties; for cond-mat the physics is quite different in spirit to the story of quasiparticles. Also inspired the study of dS/CFT correspondence where the CFT is non-unitary.
- H. Nastase published a nice textbook introducing AdS/CFT in 2015.

Holographic dictionary for $AdS_5 \times S^5$

- AdS/CFT is a strong/weak duality,

$$\frac{L_{AdS}^4}{\ell_s^4} \leftrightarrow g_{\rm YM}^2 N\,, \qquad g_s \leftrightarrow \frac{1}{N}\,.$$

  This is why it is so powerful: to learn about strongly coupled CFT we use weakly coupled string theory in AdS and vice versa.

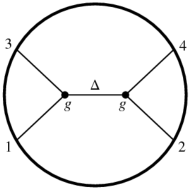
- Isometries of the bulk match up with symmetries of the CFT.
  1. The S^5 factor has a natural $SO(6)$ isometry. This is is also the R-symmetry of the $\mathcal{N}=4$ SUSY SYM theory.
  2. The AdS_5 factor has isometry group $SO(4,2)$. This is the conformal symmetry group of $\mathcal{N}=4$ $d=3+1$ SYM. $SO(3,1)$, the Lorentz group, is a subgroup.

- One of the reasons why AdS/CFT works is that near infinity, the area element grows like the volume element. The dimensionful constant of proportionality is furnished by the radius of curvature of AdS.

- Another very useful physics fact to know about AdS is that higher-ℓ partial waves do *not* fall off with higher powers of $1/r$ like they do in asymptotically Minkowski space. Influences deep in the AdS bulk can be easily seen from infinity. This is related to the fact that AdS has a timelike boundary.

Holography and black holes

- Nonlocal probes: correlation functions, Wilson loops, entanglement entropy. Witten diagrams: bulk-boundary propagators and bulk vertices.



- UV/IR relations taught us that high-energy in CFT corresponds to near-boundary in bulk. Holographic RG: running understood via bulk Hamilton-Jacobi, including counterterms. *Flows* change interior of AdS.
- A very deep fact about AdS/CFT is that it is not just a zero-temperature equivalence. Asymptotically AdS black holes correspond to turning on finite temperature in the CFT. The Hawking-Page transition from BH to hot AdS is dual to the deconfinement transition in the boundary theory.
- AdS/CFT shows us how one extra bulk spacelike dimension emerges, but generically without a path integral proof as yet. It also does not show us how time emerges. That may require going beyond QM as we currently know it.