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### HW1 Q3: Playing with $SU(3)$

*This problem is mostly about learning how to recruit a computer algebra engine to automate boring analytical calculations. Please use only Maxima (free, open source), Mathematica, or Maple. Document your code clearly so that I know you authored it.*

Consider the Lie group  $SU(3)$  pertinent to QCD. In the fundamental representation  $f$  with 3-component column vectors representing red, green and blue quarks, the generators in the standard Gell-Mann basis are

$$\begin{aligned} T^1 &= \frac{1}{2} \begin{bmatrix} 0 & +1 & 0 \\ +1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} & T^4 &= \frac{1}{2} \begin{bmatrix} 0 & 0 & +1 \\ 0 & 0 & 0 \\ +1 & 0 & 0 \end{bmatrix} & T^7 &= \frac{1}{2} \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -i \\ 0 & +i & 0 \end{bmatrix} \\ T^2 &= \frac{1}{2} \begin{bmatrix} 0 & -i & 0 \\ +i & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} & T^5 &= \frac{1}{2} \begin{bmatrix} 0 & 0 & -i \\ 0 & 0 & 0 \\ +i & 0 & 0 \end{bmatrix} & T^8 &= \frac{1}{2\sqrt{3}} \begin{bmatrix} +1 & 0 & 0 \\ 0 & +1 & 0 \\ 0 & 0 & -2 \end{bmatrix} \\ T^3 &= \frac{1}{2} \begin{bmatrix} +1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{bmatrix} & T^6 &= \frac{1}{2} \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & +1 \\ 0 & +1 & 0 \end{bmatrix}. \end{aligned} \quad (1)$$

- (a) Explain why the generators of  $su(3)$  in eq.(1) are traceless and Hermitean and why there are eight of them. Explain which ones form the Cartan subalgebra of  $su(3)$ .
- (b) In general, for any representation  $r$ ,

$$\text{Tr} \{T^A T^B\} = \mathcal{C}(r) \delta^{AB}. \quad (2)$$

Show by explicit computation from the generators in eq.(1) that  $\mathcal{C}(f) = \frac{1}{2}$ .

Do a couple of representative examples by hand to check that your code works properly, then automate the rest of the boring algebra with code.

- (c) Starting from the normalization condition  $\text{Tr} (T_r^A T_r^B) = \mathcal{C}(r) \delta^{AB}$  and the Lie algebra commutation relations  $[T^A, T^B] = i f^{ABC} T^C$  for any representation, show that

$$f^{ABC} = -\frac{i}{\mathcal{C}(r)} \text{Tr} ([T_r^A, T_r^B] T_r^C). \quad (3)$$

Use this formula to compute the structure constants  $f^{ABC}$  for  $su(3)$  using the explicit form of the generators in eq.(1).

Automate the boring algebra with code.

- (d)\* For bonus points, show that for the adjoint representation  $\mathcal{C}(Adj) = N$ .

The ingredients needed are eq.(2),  $(T_{Adj}^A)^{BC} = -i f^{ABC}$ , and code.

*Reminder: your entire assignment including code worksheets must fit within 25 pages.*

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