PHY484S/1484S GR1 (2018-19) – HW1 – due @ 10am M04Feb2019

1.3: FRW explorations

(a) In a spatially flat FRW geometry, show that the luminosity distance and angular diameter distance are given, in the small redshift limit $z \ll 1$, by

$$\begin{aligned} d_L &= \frac{1}{H_0}\left[z + \frac{1}{2}(1-q_0)z^2 + \ldots\right] , \\ d_A &= \frac{1}{H_0}\left[z - \frac{1}{2}(3+q_0)z^2 + \ldots\right] . \end{aligned} \tag{16}$$

Hence show that the angular diameter of a standard object can increase as z increases. Do these results still hold in a spatially curved FRW geometry?

(b) Using the Friedmann-Lemaître equations, show that the deceleration parameter q for our multi component cosmological fluid is given by

$$q = \frac{1}{2}\left(\Omega_m + 2\Omega_r - 2\Omega_\Lambda\right) . \tag{17}$$

Also, show that the density parameters evolve over time according to

$$\dot{\Omega}_A = \Omega_A H \left(2q - 1 - 3w_A\right) , \tag{18}$$

for $A = m, r, \Lambda$, while for the curvature case $A = k$ the evolution is

$$\dot{\Omega}_k = 2\Omega_k H q \, . \tag{19}$$

Use this to find an analytic expression for $\Omega_k(z)$ as a function of z and present day density parameters. You should get

$$\Omega_k(z) = \frac{\Omega_{k,0}}{\Omega_{m,0}(1+z) + \Omega_{r,0}(1+z)^2 + \Omega_{\Lambda,0}(1+z)^{-2} + \Omega_{k,0}} . \tag{20}$$