

**Q1: SSB for a charged adjoint scalar coupled to  $SU(5)$  Yang-Mills (PS 20.1+)**

Consider a scalar matter field  $\Phi$  transforming in the adjoint representation of a group  $G$ . Its transformation law under a group element  $U \in G$  is

$$\Phi \rightarrow \Phi' = U \Phi U^{-1}.$$

(This contrasts with fields  $\chi$  in the fundamental representation which transform as  $\chi \rightarrow U\chi$ .)

(a) Confirm that taking the covariant derivative on adjoint matter via the prescription

$$D_\mu \cdot = \partial_\mu \cdot - ig [A_\mu, \cdot]$$

gives the component form shown in lecture notes,

$$(D_\mu \Phi)^C = \partial_\mu \Phi^C + gf^{ABC} A_\mu^A \Phi^B.$$

Then, using the minimal coupling prescription for the scalar field  $\Phi$  and the assumption that it is subject to a Mexican hat self-interaction potential giving rise to spontaneous symmetry breaking, write down the Lagrangian governing the dynamics of  $\Phi$ .

(b) Specialize to the group<sup>1</sup>  $SU(5)$ . What kind of matrix must  $\Phi$  be? Assuming that  $\Phi$  develops a vev

$$\langle \Phi \rangle_1 = A \text{diag}(+1, +1, +1, +1, -4),$$

determine the unbroken symmetry group left over after SSB. Be particularly careful to explain where any  $U(1)$ s come from. Count how many massless and massive degrees of freedom there are before and after SSB, for scalar and vector modes.

(c) Repeat part (b) with a vev of the form

$$\langle \Phi \rangle_2 = B \text{diag}(+2, +2, +2, -3, -3).$$


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(PLEASE TURN OVER)  $\longrightarrow$

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<sup>1</sup>This group has interested model builders because it can be used to unify quarks and leptons.

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**Q2: Scalars, sources and Yukawa potentials** (Banks 3.2)

Compute the generating functional for a free scalar field of mass  $m$  in  $3 + 1$  dimensions, in the presence of a source satisfying

$$J(t, x) = \begin{cases} q_1 \delta^3(x) + q_2 \delta^3(x - R), & |t| < T \\ 0, & |t| > T \end{cases}$$

This source causes a static disturbance of the field, at two spatial points, over a time interval  $2T$ . When  $T \gg R \gg 1/m$ , the amplitude should have the form  $\exp\{-2i V(R) T\}$ , where  $V(R)$  is the lowest energy of states with two localized disturbances. Show that this is the case and that (up to an additive constant)  $V$  is the Yukawa potential.

Thus, particle exchange is responsible for forces between static disturbances. This motivates the claim that all particle interactions can be understood in terms of particle creation, annihilation, and exchange.

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**Q3: Four-legged Legendre trees for scalar fields**

In a lecture, we explained the origin of the general formula relating connected Green's functions  $\{W_n\}$  to one-particle irreducible vertices  $\{\Gamma_n\}$  and complete propagators  $W_2$ . In particular, we derived the three-point formula.

Derive the four-point formula:

